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A multi-objective robust optimization model for logistics planning in the earthquake response phase

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ABSTRACT

Usually, resources are short in supply when earthquakes occur. In such emergency situations, disaster relief organizations must use these scarce resources efficiently to achieve the best possible emergency relief. This paper therefore proposes a multi-objective, multi-mode, multi-commodity, and multi-period stochastic model to manage the logistics of both commodities and injured people in the earthquake response. Also, a robust approach is developed and used to make sure that the distribution plan performs well under the various situations that can follow an earthquake. Afterwards, it proposes a solution methodology according to hierarchical objective functions and uses it to illustrate the customized robust modeling approach.

1. Introduction

In the literature several definitions for disasters have been proposed. The most widely accepted definition of disaster is the one provided by the World Health Organization. According to this definition, a disaster is any occurrence that brings about damages, destruction, ecological disruption, loss of human life, human suffering, deterioration of health and health service on a scale adequate to warrant an extraordinary response from outside the affected area (Barbarosoglu and Arda, 2004). One of the mentioned occurrences is an earthquake, which often causes huge property damages, human injuries and casualties. Increase of this type of disaster over the last years (Eshghi and Larson, 2008) along with growth of population density in areas sensitive to earthquakes, create a growing need for designing emergency response procedures prior to an earthquake disaster. These emergency procedures guide the set of actions taken during the initial phase of this emergency situation, the so-called "Earthquake Response Phase".

Generally, the two most important intervention activities during earthquake response are the evacuation of people and the logistics of materials. Evacuation takes place during the initial phase of the emergency response phase to extricate the injured and casualties from the area. Logistics activities continue for a longer period of time as they aim to provide the necessary disaster relief commodities to people in the affected areas, and transport the injured people to the hospitals or the emergency medical centers within the affected area. An efficient planning of logistical activities during the earthquake response phase can therefore tremendously decrease the loss of human life in the event of an earthquake. As a result, many

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researchers have focused on this subject since the late 1980s. Most of his research considers the logistics of a single type of commodity or of injured people. For instance, Knott (1987) proposed a linear programming model for the bulk food transportation problem to minimize the transportation cost, maximize the amount of food delivered. Furthermore, a linear programming model to determine the vehicle schedules for transporting the bulk food to a disaster area is presented in Knott (1988).

Oh and Haghani (1996, 1997) analyze the transportation of different disaster relief commodities such as food, clothing, medicine, medical supplies, machinery and personnel to minimize the loss of life and maximize the efficiency of the rescue operations. The authors formulate a multi-commodity, multi-modal network flow models for generic disaster-relief operations. Other commodity logistics planning models are provided by Barbarosoglu et al. (2002), Ozdamar et al. (2004), Tzeng et al. (2007), Sheu (2007, 2010), Nolz et al. (2011), Lin et al. (2011), Zhan and Liu (2011), Afshar and Haghani (2012), and Zhang et al. (2012). Barbarosoglu et al. (2002) focus on the use of helicopters for aid delivery and rescue missions during natural disasters. The authors use existing research on the helicopter routing to address crew assignment, routing and transportation issues during the initial response phase of disaster management. Ozdamar et al. (2004) present a network-based multi-period model to plan the commodity logistics in the natural disaster response. The model first determines the amount of commodities to be transported between two adjacent nodes in the network. Then, another algorithm uses these amounts to determine the origin and destination of commodities transporting in the networks. Tzeng et al. (2007) develop a multi-objective relief-distribution model for designing real-life relief delivery systems. The model features three objectives, including minimization of the total cost, minimization of the total travel time, and maximization of the minimal satisfaction during the planning period. Sheu (2007) presents a hybrid fuzzy clustering-optimization approach to coordinate the relief logistics flows in a three-layer relief supply network during the crucial rescue period. The proposed approach involves two recursive mechanisms (disaster-affected area grouping and relief co-distribution) in a network with relief suppliers, urgent relief distribution centers, and relief demanding areas. Sheu (2010) also presents a dynamic relief-demand management model for emergency logistics operations under imperfect information conditions in large-scale natural disasters. This model consists of three main steps: data fusion to forecast relief demand in multiple areas, fuzzy clustering to classify affected area into groups, and multi-criteria decision making to rank the order of priority of groups. Nolz et al. (2011) develop a multi-objective model for relief aid distribution for a post-natural-disaster situation. This model encompasses three objective functions, including minimizing the risk, maximizing the coverage provided by the logistics system and minimizing the total travel time. Moreover, Lin et al. (2011) propose a multi-item, multi-vehicle, multi-period and multi-objective model for delivery of prioritized items in disaster-relief operations. This model includes two objective functions, which minimize the total unsatisfied demands and the total travel time for all tours and all vehicles. Zhan and Liu (2011) present a multi-objective stochastic programming model to handle the uncertainty of demand, supply and the availability of transportation paths in an emergency logistics network. The model focuses on minimizing the expected travel time and the proportion of unmet demands by using chance constraints and scenario planning. Afshar and Haghani (2012) propose a mathematical model to control the flow of several relief commodities in the response network. This model considers the optimal locations for several layers of temporary facilities, routing and pick up or delivery schedules. Finally, Zhang et al. (2012) propose an integer mathematical model to allocate the available resources to demand points subject to constraints on multiple resources and possible secondary disasters. This model minimizes the cost of the total time of dispatching emergency resources.

Another stream of research mainly focused on transport of injured people, such as Fiedrich et al. (2000) and Jotshi et al. (2009). Fiedrich et al. (2000) present a model for allocating resources in an earthquake response phase to handle the logistics of injured persons. Moreover, Jotshi et al. (2009) develop a robust methodology for the dispatching and routing of emergency vehicles in a post-disaster environment.

Although most of the previous papers examine a single type of logistical activity, some have addressed both disaster relief commodity and injured people logistics during the earthquake response phase. Yi and Ozdamar (2007) present a dynamic logistics coordination model for evacuation during the disaster response phase. This model investigates flows of both commodities and wounded people, and minimizes the sum of unserved injured persons and unsatisfied commodity demand. Also, Yi and Kumar (2007) present a model to schedule the dispatching of commodities to distribution centers in the affected areas and for transporting the injured persons from the affected areas to the medical centers. Ozdamar (2011) propose a mathematical model to transport injured people and medical items such as medicine and vaccines to the affected locations by helicopter. This model aims to minimize the total mission time required to complete the transportation tasks. Finally, Ozdamar and Demir (2012) present a hierarchical cluster and route procedure for coordinating vehicle routing in large-scale post-disaster distribution and evacuation activities.

Most of the information received at the disaster management center – such as the number of injured people, the amount of demands, network situation, available commodities and hospital's capacities – is imprecise and uncertain. To take this uncertainty into account, Barbarosoglu and Arda (2004), Ma et al. (2010), Jotshi et al. (2009) and Zhan and Liu (2011) use stochastic modeling techniques. Zhan and Liu (2011) consider the uncertainty of demand, supply and the availability of paths in a location-allocation problem, and use chance constraints, scenario planning and goal programming to handle these uncertainties. Moreover, Jotshi et al. (2009) use scenario planning to consider the uncertainty of damages and available network. Ma et al. (2010) present a min–max robust multi-point, multi-vehicle transportation model to minimize the maximum rescue time for moving injured people. In the model, it is assumed that the distances between affected area and medical centers are uncertain. Scenario planning is used represent the data uncertainty. Barbarosoglu and Arda (2004) develop a two-stage stochastic programming framework for minimizing the expected transportation cost in the disaster response phase.

They also use scenario planning-but in two stages-to model data uncertainty. In the first stage, a limited number of Earthquake Scenarios (ES) are considered to determine the magnitude of earthquake. Along the same lines, few impact scenarios (IS) specific possible impacts of the earthquake in each ES. More details about the models discussed above are summarized in Table 1.

Table 1 classifies the models according to twelve criteria classified in three classes which are modeling, transportation and flow. The first class includes type of modeling, type of objective function, number of objective functions, solution methodology, routing, and optimization method. According to the first criterion, papers are categorized into two groups: linear programming (LP) and non-linear programming (NLP). Moreover, in the second criterion, papers are categorized into three groups including cost objective function (C), humanitarian objective function (H), and both type of cost and humanitarian (CH). The third criterion, number of objective functions, classifies the papers into two groups: single objective (SO) and multi objectives (MO). The forth criterion, solution methodology, also categorizes the paper into three groups including heuristics (H), meta-heuristics (MH) and exact methodology (Ex). The next criterion considers the routing ability of the proposed models. According to this criterion, papers are categorized into two groups. The first group consists of those papers for which the routes are presumed to be known (PR), and the second group is those papers in which the routes are to be determined by the model (R). Finally, the optimization method classifies the papers into four groups: deterministic optimization (DtO), stochastic optimization (StO), robust optimization (RbO) and fuzzy optimization (FuO).

The second class of criteria, transportation, includes three criteria which are mode of transportation, combined transportation, and available vehicles. According the first criterion of this group, papers are divided into single mode (SM) and multi mode (MM) models. The second criterion, combined transportation, investigates the capability of combined transportation in the proposed models (combined transportation (TV) and models do not consider the combined transportation (SV)). Finally, the third criterion considers the condition of available vehicles in the network. According to this criterion, papers are categorized into three groups. The first group assumes that the number of vehicles is known (FV), the second group presumes that the number of vehicles is uncertain (UV), and the last group assumes that there is no restriction on the number of available vehicles (UV). In addition, the third class of criteria includes three criteria: flow type, supply type and demand type. Flow type considers the number of different flows in the supply network. According to this criterion, papers are classified into a group that considers a single commodity or injured people for transportation (SF), and the second group that considers both commodities and wounded people for transportation (MF). The second and third criteria respectively investigate type of supply and demand in the network. According to the second criterion, papers are categorized into two groups. One group assumes that the amounts of available supplies are deterministic (DS) and another group presumes that the amounts of available supplies are uncertain (SS). Finally, the third criterion classifies the papers into two groups: deterministic demand (DD) and uncertain demand (SD).

Although logistics planning during earthquake response is clearly receiving increased attention in the literature, the existing models exhibit a number of drawbacks. First, Barbarosoglu and Arda (2004), Jotshi et al. (2009) and Ma et al. (2010) use scenario planning to model uncertainty during disaster response. Although this approach is obviously superior to a deterministic approach which ignores the uncertainty which is clearly present in practice, it is not a very efficient approach for tackling real world problems. In practice, uncertain data usually changes in a set. Therefore, modeling approaches based on uncertainty sets are more appropriate and more precise than scenario-based ones. Second, most of the existing models only consider a single commodity type or make no distinction between different levels of injuries when planning logistics relief efforts. Furthermore, the few papers which do consider both relief commodities and injured logistics (e.g. Yi and Ozdamar, 2007; Yi and Kumar, 2007; Ozdamar, 2011; Ozdamar and Demir, 2012), often do not take transportation restrictions into account, and presume that all vehicles could transport all types of commodities and injured people (Yi and Ozdamar, 2007; Yi and Kumar, 2007). Fourth, existing research does not consider the priorities and hierarchy of response objectives. Generally, three types of objectives have been considered in the earthquake response. These objective functions are minimization of transportation cost, minimization of unsatisfied demands and minimization of unserved injured people. These three objectives clearly do not have the same priority. Naturally, serving the injured people and transporting them to the hospitals or emergency medical centers is more important than satisfying the demands for disaster relief commodities. Moreover, minimization of transportation cost is significant after minimization of these objectives. It is worth noting that, these priorities may change in some exceptional conditions in which failing to satisfy commodity demand would cause additional casualties. Finally, none of the previous proposed models considered the combined transportation in its planning.

To overcome these drawbacks, this paper first develops a robust approach for stochastic model with uncertain right-hand sides based on the approach proposed by Bertsimas and Sim (2004). Moreover, this paper presents a multi-objective, multi-mode, multi-commodity, multi-period stochastic model to manage both relief commodities and injured people logistics in the initial phase of earthquake response. Furthermore, the model takes into account combined transportation, vehicle capabilities and represents the data uncertainty by interval data. The proposed stochastic model has three hierarchical objective functions which respectively are: minimization of total (weighted) waiting time of unserved injured persons, minimization of total (weighted) lead time of meeting the commodity needs, and minimization of total vehicles utilized in the response. Finally, this paper applies the proposed robust approach for managing relief commodities and injured people in the initial phase of earthquake response.

Note that Yi and Ozdamar (2007) and Barbarosoglu and Arda (2004) address similar research objectives. However, our paper is significantly different as the dynamic model proposed by Yi and Ozdamar is deterministic. Furthermore, although this model plans both relief commodities and injured logistics activities, it considers no restriction on the transportation of

Table 1
Properties of disaster management models.

Reference	Year	Modeling						Transportation			Flow		
		Type of modeling	Type of objective function	# Objective functions	Solution methodology	Routing	Optimization method	Mode of transportation	Combined transportation	Available vehicles	Flow type	Supply type	Demand type
Oh S. and Haghani	1996	LP	C	SO	Hu	R	DtO	MM	TV	FV	MF	FS	FD
Oh S. and Haghani	1997	LP	C	SO	Hu	R	DtO	MM	TV	FV	MF	FS	FD
Fiedrich et al.	2000	LP	H	SO	MH	R	DtO	SM	SV	FV	SF	FS	FD
Barbarosoglu et al.	2002	LP	CH	MO	Hu	R	DtO	SM	SV	FV	MF	FS	FD
Barbarosoglu and Arda	2004	LP	C	SO	Ex	R	StO	MM	TV	SV	SF	SS	SD
Ozdamar et al.	2004	LP	H	SO	Ex	R	DtO	MM	TV	FV	SF	DS	DD
Tzeng et al.	2006	LP	CH	MO	Hu	PR	DtO	SM	SV	UV	SF	FS	FD
Yi and Ozdamar	2007	LP	H	SO	Hu	R	DtO	SM	TV	FV	MF	FS	FD
Yi and Kumar	2007	LP	H	SO	MH	R	DtO	SM	TV	FV	MF	FS	FD
Sheu	2007	LP	C	SO	Hu	PR	FuO	SM	SV	FV	SF	FS	SD
Jotshi et al.	2009	LP	H	SO	Hu	R	RbO	SM	SV	FV	SF	FS	SD
Ma et al.	2010	NLP	H	SO	Ex	PR	RbO	SM	SV	FV	SF	FS	FD
Nolz et al.	2011	LP	H	MO	Hu	R	DtO	SM	SV	FV	SF	FS	FD
Lin et al.	2011	LP	H	SO	Hu	R	DtO	SM	SV	FV	SF	FS	FD
Ozdamar	2011	LP	H	MO	Ex	PR	DtO	SM	SV	FV	MF	FS	FD
Zhan and Liu	2011	LP	H	MO	Ex	R	DtO	SM	SV	FV	SF	SS	SD
Ozdamar and Demir	2012	LP	H	SO	Hu	R	DtO	MM	SV	FV	MF	FS	FD
Afshar and Haghani	2012	LP	H	SO	Ex	R	DtO	MM	TV	FV	SF	FS	FD
Zhang	2012	LP	C	SO	Hu	PR	DtO	SM	SV	UV	SF	FS	FD

available disaster relief commodities and injured people. In addition, this model assumes that the requests for satisfying commodity demand and serving injured people are equivalent. Therefore, it minimizes the sum of total unsatisfied commodity needs and total number of unserved injured people. In practice, these two types of requests clearly have different priorities. Furthermore, the model proposed by Barbarosoglu and Arda uses a two-stage scenario planning approach to represent data uncertainty whereas uncertainty sets are more common in practice. As a result, obtaining the appropriate scenarios often proves to be difficult. Moreover, none of these researches considers the hierarchy of objective functions or supports the combined transportation of people and relief materials.

The remainder of this paper is organized as follows. The second section defines the planning problem during earthquake response and discusses common modeling assumptions in more detail before presenting a Stochastic Model for Logistics Management (SMLM) to schedule the logistical activities in the initial phase of earthquake response. Given that existing robust optimization approaches from the literature are not appropriate for the proposed model, the robust optimization approach of Bertsimas and Sim (2004) is revised in Section 3. Section 4 uses the adjusted robust approach from Section 3 to obtain the robust counterpart of the proposed stochastic model, and convert it into an equivalent deterministic model. Section 5 proposes a solution methodology to solve the robust model obtained in Section 4. Finally, an illustrative example and conclusion and future researches are presented in Sections 6 and 7 respectively.

2. Problem definition

In general, the logistics plan for the disaster response phase has two objectives. The first objective is transporting the injured people from the affected area to the hospitals or other emergency medical centers, and the second one is dispatching the necessary disaster relief commodities from predefined warehouses or suppliers to the affected area. In these circumstances, available resources, commodities and vehicles are usually inadequate. Therefore, a manager should have an effective and efficient plan to achieve his objectives as much as possible. Even if there would be no shortages, e.g. because of the limited damage caused by a minor earthquake, managers will still benefit from an optimization model to allocate the available resources in the best possible way.

In addition to the complexity in handling of the logistics activities, the available network data on demands, suppliers and hospitals are usually uncertain during the planning horizon of a large-scale earthquake relief effort. These uncertainties further complicate the planning of logistical activities for disaster relief organizations. Therefore, this paper aims at developing a multi-objective, multi-mode, multi-commodity, multi-period stochastic model to manage both disaster relief commodities and the logistics of injured people in the initial phase of earthquake response effort. To determine the other features of the model, the factors from Table 1 are used to design different scenarios that mimic real-life conditions in Table 2. Finally in the last column, the most practical model features are chosen for developing the logistics model.

Disaster relief managers want to minimize the total (weighted) waiting time of unserved injured people, the total (weighted) lead time before satisfying commodity requests and the number of vehicles deployed in the response effort. Without loss of generality, we want to give the highest priority to servicing the injured, the second highest priority to shipping the disaster relief commodities, and the lowest priority to the minimization of vehicles used in response efforts. Hence, the proposed model, Stochastic Model for Logistics Management (SMLM), minimizes these objectives are optimized hierarchically or lexicographically (see e.g. Marler and Arora, 2003) because of their differing importance levels. Other main assumptions of the proposed model are as follows:

Table 2
Different scenarios of identified factors.

Criteria	Scenarios for real-life condition			Selected scenario
	Scenario 1	Scenario 2	Scenario 1	
Modeling				
Type of modeling	LP	NLP	–	*LP*
Type of objective function	CH	H	C	*CH*
# Objective functions	MO	SO	–	*MO*
Solution methodology	Ex	MH	Hu	*Ex*
Routing	R	PR	–	*R*
Optimization method	RbO/StO	FuO	DtO	*RbO*
Transportation				
Mode of transportation	MM	SM	–	*MM*
Combined transportation	TV	SV	–	*TV*
Available vehicles	FV	SV	UV	*FV*
Flow				
Flow type	MF	SF	–	*MF*
Supply type	SS	DS	FS	*SS*
Demand type	SD	DD	FD	*SD*

- Demand nodes, supply nodes, emergency medical centers or hospitals and the distance between them are known.
- The number of affected areas and the corresponding transportation arcs are given. In other words, the model assumes that information on the transport infrastructure is available via advanced disaster detection technologies such as satellites and geographic information system (GIS).
- There are several modes of transport with various types of vehicles having different capacities.
- There are several types of injuries with different priorities. Also, the injured need several types of commodities and drugs whose priorities can be defined by the model user. Note that, the number of types of injuries (Fiedrich et al., 2000), types of commodities and their priorities can e.g. be determined based on several external criteria such as the population affected, the magnitude of earthquake and etc.
- Any vehicle permitted to transport the relief commodities can carry them from one or multiple supply nodes to one or multiple demand nodes. In addition, any vehicle authorized to transport the injured persons can transport them from one or multiple affected nodes to one or multiple emergency medical centers or hospitals.
- Each vehicle can transport pre-specified types of commodities or injured people. In other words, a vehicle cannot transport all types of commodities and injured people.
- No vehicle can carry both commodities and injured people simultaneously.
- Commodities and injured people have to be directed moved from their pickup to their destination location without intermediate stops, but can be transshipped into other appropriate vehicles at intermediate stops. Moreover, it is presumed that this transmission does not take any time.
- The weight capacity and the volume capacity of each vehicle carrying commodity are known. Moreover, the capacity of the vehicle carrying the wounded people is known too.
- There are several types of injuries and commodity demands with different priorities.
- The number of injured people, the amount of commodity demands, suppliers' capacities and hospitals' capacities in the planning horizon are uncertain. However, they can be estimated for the next periods based on the magnitude of earthquake, properties of affected area, available data in disaster management center and experts' previous experiences. The uncertain data is presented by an uncertain set defined by a nominal value and a permitted change. In addition, these sets can be unequal for different periods as they can follow from non-identical distributions.
- Since vehicles are often not been suitably equipped to treat the injured, injured persons are considered serviced when delivered to a hospital or an emergency medical center. In other words, a served person is not serviced when assigned to a vehicle but when delivered to a hospital or an emergency medical center.

Also, the parameters used in the SMLM model are as follows:

T	length of the planning horizon,
N	set of all nodes in the network, $R = \|N\|$
DN	set of demand nodes, $DN \subset N$ and $M = \|DN\|$
SN	set of supplier nodes, $SN \subset N$ and $L = \|SN\|$
HN	set of hospital nodes, $HN \subset N$ and $Q = \|HN\|$
IN	set of intermediate nodes, the nodes which are not a supply, demand or hospital node, $IN \subset N$ and $I = \|IN\|$
KN	set of all nodes except demand nodes, $KN = N \backslash \{DN\}$
NN	set of all nodes except hospital nodes, $NN = N \backslash \{HN\}$
CS	set of commodities types, $A = \|CS\|$
VS	set of vehicle types, $V = \|VS\|$
IS	set of injured people types, $H = \|IS\|$
t,s	denote time stamps in the planning horizon
m	denotes the nodes affected by the earthquake
l	denotes a specific node in which supplier exists
q	denotes a specific node in which hospital exists
i	denotes a specific node in IN set
k	denotes a specific node in KN set
n	denotes a specific node in NN set
o,p	index of set N
a	denotes a specific commodity
v	denotes a specific vehicle
h	denotes a specific injury type
M_{Big}	a large positive number
$\tilde{d}_{amt}$	amount of commodity type a demanded at node m at time t
$\tilde{w}_{hmt}$	number of injured persons type h evacuated at node m at time t
$s\tilde{u}p_{als}$	amount of commodity type a supplied at node l at time s
$surp_{amt}$	amount of surplus commodity type a exist at node m at time t

t^{v}_{op}	time required for traversing arc (o,p) by vehicle type v
av^{v}_{ot}	available vehicle type v at node o at time t
$w\tilde{s}p_{hqt}$	capacity of the hospital located at node q for treating injury type h at time t
w_a	unit weight of commodity type a
c_a	unit volume of commodity type a
cw^{v}	weight capacity of vehicle type v
cc^{v}	volume capacity of vehicle type v
dv^{v}	capacity of vehicle type v
p_a	priority for satisfying demand of commodity type a
p'_h	priority for servicing injured people type h

$$\delta^{v}_{sopt} = \begin{cases} 1 & \text{If vehicle type } v \text{ departing node } o \text{ at time } s \text{ arrives at node } p \text{ before time } t+1 \\ 0 & \text{otherwise}, \end{cases}$$

$$ac^{v}_{a} = \begin{cases} 1 & \text{if vehicle type } v \text{ be able to carry commodity type } a, \\ 0 & \text{otherwise}, \end{cases}$$

$$aw^{v}_{h} = \begin{cases} 1 & \text{if vehicle type } v \text{ be able to carry injured people type } h, \\ 0 & \text{otherwise}. \end{cases}$$

Finally, the following variables are defined for the SMLM model.

dev_{amt}	amount of unsatisfied demand of commodity type a at node m at time t
dew_{hmt}	number of injured people type h not serviced at node m at time t
Z^{v}_{opt}	number of vehicle type v moving from node o to node p at time t
X^{lv}_{aopt}	amount of commodity type a belonging to supplier located at node l, and dispatched from node o to node p at time t by vehicle type v
Y^{mv}_{hopt}	number of injured people of type h belonging to the affected node m,and dispatched from node o to node p at time t by vehicle type v
avv^{v}_{ot}	number of available vehicle type v at node o at time t
TF^{v}_{ait}	amount of commodity type a transferred from the vehicle type $v$to other types of vehicle at node i at time t
TT^{v}_{ait}	amount of commodity type a transferred from other types of vehicles to the vehicle type v at node i at time t
TFD^{v}_{hit}	number of injured people type h transferred from the vehicle type v to other types of vehicles at node i at time t
TTD^{v}_{hit}	number of injured people type h transferred from other types of vehicles to the vehicle type v at node i at time t

Because it can be proven that the total (weighted) waiting time of injured people and the total (weighted) lead time of unsatisfied demand are respectively equivalent to the total (weighted) number of injured person-periods not serviced, and the summation of unsatisfied commodity needs during the planning horizon (Najafi et al., submitted for publication), the SMLM model uses these equivalents in its objective functions for sake of simplicity. The SMLM model can therefore be stated as follows:

$$\text{Min} \quad f_1 = \sum_{h=1}^{H}\sum_{m=1}^{M}\sum_{t=1}^{T} P'_h \cdot dew_{hmt}, \tag{1}$$

$$\text{Min} \quad f_2 = \sum_{a=1}^{A}\sum_{m=1}^{M}\sum_{t=1}^{T} P_a \cdot dev_{amt}, \tag{2}$$

$$\text{Min} \quad f_3 = \sum_{o=1}^{O}\sum_{p=1}^{P}\sum_{t=1}^{T} Z_{opt}, \tag{3}$$

$$\text{S.t.}: \quad \sum_{v=1}^{V}\sum_{l=1}^{L}\sum_{s=1}^{t}\sum_{o=1}^{O} X^{lv}_{aoms} \cdot \delta^{v}_{somt} - \sum_{v=1}^{V}\sum_{l=1}^{L}\sum_{s=1}^{t}\sum_{p=1}^{P} X^{lv}_{amps} - \sum_{s=1}^{t} \tilde{d}_{ams} = surp_{amt} - dev_{amt} \quad \forall m \in DN,\ a \in CS,\ t \in T, \tag{4}$$

$$-\sum_{v=1}^{V}\sum_{s=1}^{t}\sum_{q=1}^{Q}\sum_{o=1}^{O} Y^{mv}_{hoqs} \cdot \delta^{v}_{soqt} + \sum_{v=1}^{V}\sum_{s=1}^{t}\sum_{q=1}^{Q}\sum_{p=1}^{P} Y^{mv}_{hqps} + \sum_{s=1}^{t} \tilde{w}_{hms} = dew_{hmt} \quad \forall m \in DN,\ h \in IS,\ t \in T, \tag{5}$$

$$\sum_{v=1}^{V}\sum_{s=1}^{t}\sum_{p=1}^{P} X^{lv}_{alps} \leqslant \sum_{s=1}^{t} s\tilde{u}p_{als} \quad \forall a \in CS,\ l \in SN,\ t \in T, \tag{6}$$

$$\sum_{v=1}^{V}\sum_{s=1}^{t}\sum_{p=1}^{P} Y^{mv}_{hmps} \leqslant \sum_{s=1}^{t} \tilde{w}_{hms} \quad \forall m \in DN,\ h \in IS,\ t \in T, \tag{7}$$

$$\sum_{v=1}^{V}\sum_{o=1}^{O}\sum_{s=1}^{t}X_{aoks}^{lv}\cdot\delta_{sokt}^{v}-\sum_{v=1}^{V}\sum_{p=1}^{P}\sum_{s=1}^{t}X_{akps}^{lv}=0\quad\forall a\in CS,\ k\in KN,\ l\in SN,\ t\in T,\ k\neq l,\tag{8}$$

$$\sum_{v=1}^{V}\sum_{o=1}^{O}\sum_{s=1}^{t}Y_{hons}^{mv}\cdot\delta_{sont}^{v}-\sum_{v=1}^{V}\sum_{p=1}^{P}\sum_{s=1}^{t}Y_{hnps}^{mv}=0\quad\forall h\in IS,\ n\in NN,\ m\in DN,\ t\in T,\ n\neq m,\tag{9}$$

$$\sum_{l=1}^{L}\sum_{o=1}^{O}\sum_{p=1}^{P}X_{aopt}^{lv}\leqslant M_{Big}\cdot ac_{a}^{v}\quad\forall a\in CS,\ v\in VS,\ t\in T,\tag{10}$$

$$\sum_{m=1}^{M}\sum_{o=1}^{O}\sum_{p=1}^{P}Y_{hopt}^{mv}\leqslant M_{Big}\cdot aw_{h}^{v}\quad\forall h\in IS,\ v\in VS,\ t\in T,\tag{11}$$

$$\sum_{o=1}^{O}\sum_{l=1}^{L}\sum_{s=1}^{t}X_{aois}^{lv}\left(\delta_{soit}^{v}-\delta_{soi(t-1)}^{v}\right)-\sum_{l=1}^{L}\sum_{p=1}^{P}X_{aipt}^{lv}=TF_{ait}^{v}-TT_{ait}^{v}\quad\forall a\in CS,\ i\in IN,\ t\in T,\ v\in VS,\tag{12}$$

$$\sum_{o=1}^{O}\sum_{m=1}^{M}\sum_{s=1}^{t}Y_{hois}^{mv}\left(\delta_{soit}^{v}-\delta_{soi(t-1)}^{v}\right)-\sum_{m=1}^{M}\sum_{p=1}^{P}Y_{hipt}^{mv}=TFD_{hit}^{v}-TTD_{hit}^{v}\quad\forall h\in IS,\ i\in IN,\ t\in T,\ v\in VS,\tag{13}$$

$$\sum_{l=1}^{L}\sum_{a=1}^{A}X_{aopt}^{lv}\cdot c_{a}\leqslant Z_{opt}^{v}\cdot cc^{v}\quad\forall o\in N,\ p\in N,\ t\in T,\ v\in VS,\tag{14}$$

$$\sum_{l=1}^{L}\sum_{a=1}^{A}X_{aopt}^{lv}\cdot w_{a}\leqslant Z_{opt}^{v}\cdot cw^{v}\quad\forall o\in N,\ p\in N,\ t\in T,\ v\in VS,\tag{15}$$

$$\sum_{m=1}^{M}\sum_{h=1}^{H}Y_{hopt}^{mv}\leqslant Z_{opt}^{v}\cdot dv^{v}\quad\forall o\in N,\ p\in N,\ t\in T,\ v\in VS,\tag{16}$$

$$Z_{opt}^{v}\leqslant M_{Big}\cdot t_{op}^{v}\quad\forall o\in N,\ p\in N,\ t\in T,\ v\in VS,\tag{17}$$

$$\sum_{o=1}^{O}\sum_{s=1}^{S}Z_{ops}^{v}\cdot\delta_{sopt}^{v}+\sum_{s=1}^{t}av_{ps}^{v}=avv_{pt}^{v}+\sum_{o=1}^{O}\sum_{s=1}^{t}Z_{pos}^{v}\quad\forall p\in N,\ t\in T,\ v\in VS,\tag{18}$$

$$\sum_{v=1}^{V}\sum_{m=1}^{M}\sum_{o=1}^{O}\sum_{s=1}^{t}Y_{hoqs}^{mv}\cdot\delta_{soqt}^{v}-\sum_{v=1}^{V}\sum_{m=1}^{M}\sum_{p=1}^{P}\sum_{s=1}^{S}Y_{hqps}^{mv}\leqslant\sum_{s=1}^{t}w\tilde{s}p_{hqs}\quad\forall h\in IS,\ q\in HN,\ v\in VS,\tag{19}$$

$$\begin{aligned}&Y\geqslant 0\ \&\ Integer,\quad X\geqslant 0\ \&\ Integer,\quad Z\geqslant 0\ \&\ Integer,\\&dev\geqslant 0\ \&\ Integer,\quad dew\geqslant 0\ \&\ Integer,\quad avv\geqslant 0\ \&\ Integer,\end{aligned}\tag{20}$$

where Eq. (1) minimizes the total (weighted) unserved injured people, and Eq. (2) minimizes the total (weighted) unsatisfied demands during the planning horizon. In addition, Eq. (3) minimizes the total vehicles utilized in the response. Note that, these equations are minimized hierarchically. Constraints (4) and (5) determine unsatisfied commodities demand and unserved injured people at demand nodes respectively. Constraint (6) ensures that the dispatched commodities are not larger than the at hand commodities of current suppliers. Constraint (7) guarantees that the amount of injured people dispatched from a specific demand node is larger than the number of evacuated people from that node. Constraints (8) and (9) enforce material flow and injured people flow on network nodes. In addition, these equations guarantee that no commodity or injured people abide in the intermediate nodes. Constraints (10) and (11) ensure that all commodities and injured people are transported by authorized vehicles. Since combined transportation is allowed, constraints (12) and (13) define transmitted commodities and injured people among permitted vehicles, respectively. Constraints (14) and (15) restrict the commodities' quantity by weight capacity and volume capacity of the used vehicles. Similarly, constraint (16) restricts the number of injured people by the vehicle capacity. Constraint (17) restricts the itinerary of each vehicle type to existing arcs. Constraint (18) balances the flow of vehicles over each node, and constraint (19) restricts the number of injured people dispatched to the hospitals to their allowed capacity. Finally, constraint (20) defines the variables. Note that the proposed model presumes that information about the transportation network is easily accessible in real time via advanced disaster detection technology. If such information is not accessible for planning, the responder could solve the model for likely network scenarios. However, these scenarios often result in different logistics plans and emergency routes for response, and a schedule prepared for one scenario mostly is inapplicable for another scenario.

3. The robust LP model with uncertain right-hand side

Robust optimization is one of the predominant approaches to solving linear optimization problems with uncertain data. The first step in this direction was taken by Soyster (1973). In this study, he proposed a linear optimization model to construct a solution that is feasible for all data that belong to a convex set. This approach was further developed by Ben-Tal and Nemirovski (1998, 1999, 2000), El-Ghaoui and Lebret (1997), El-Ghaoui et al. (1998), Bertsimas and Sim (2004). Bertsimas and Sim (2004) propose a solution approach for a linear mathematical model with an uncertain coefficient matrix. Their approach provides a robust solution whose level of conservatism can be flexibly adjusted in terms of probabilistic bounds for constraint violation. Since the SMLM model has a deterministic zero–one coefficient matrix like most of the logistics models, the method proposed in Bertsimas and Sim (2004) cannot provide its robust counterpart. Therefore, this section aims at explaining and customizing the Bertsimas and Sim (2004) approach for a robust optimization of the SMLM model. It is worth noting that this customized approach can be used for all logistics models in which right-hand side is a summation of some uncertain parameters.

Bertsimas and Sim (2004) consider the following model,

$$\begin{aligned} \text{Max}\quad & z = \mathbf{cx}, \\ & \tilde{\mathbf{a}}\mathbf{x} \leqslant \mathbf{b}, \\ & \mathbf{l} \leqslant \mathbf{x} \leqslant \mathbf{u}, \end{aligned} \tag{21}$$

in which some parameters of the coefficient matrix (a_{ij}) are uncertain. In addition, each uncertain parameter ($\tilde{a}_{ij}$) takes a value according to a symmetric distribution with mean equals to the nominal value (a_{ij}) in the interval $[a_{ij} - \hat{a}_{ij}, a_{ij} + \hat{a}_{ij}]$. Furthermore, they define a parameter Γ_i for every constraint. This parameter is not necessarily integer and takes a value in the interval $[0, |J_i|]$, J_i being the set of uncertain parameters in the ith constraint). Finally, they propose a linear robust counterpart to protect against all cases that $\lfloor \Gamma_i \rfloor$ coefficients of set J_i are permitted to change, and one coefficient (a_{it_i}) changes by $(\Gamma_i - \lfloor \Gamma_i \rfloor)\hat{a}_{it_i}$. To guarantee feasibility, they consider a protective function for every constraint i which are named $\beta(\mathbf{x}, \Gamma_i)$ and are equal to

$$\beta(\mathbf{x}, \Gamma_i) = \underset{\{S_i \cup t_i | S_i \subseteq J_i, |S_i| = \lfloor \Gamma_i \rfloor, t_i \in J_i \setminus S_i\}}{Max} \left\{ \sum_{j \in J_i} \hat{a}_{ij} |x_j| + (\Gamma_i - \lfloor \Gamma_i \rfloor) a_{it_i} |x_{t_i}| \right\}. \tag{22}$$

Therefore, model (21) can be rewritten as model (23)

$$\begin{aligned} \text{Max}\quad & z = \mathbf{cx}, \\ \text{s.t.}:\quad & \sum_j a_{ij} x_j + \underset{\{S_i \cup t_i | S_i \subseteq J_i, |S_i| = \lfloor \Gamma_i \rfloor, t_i \in J_i \setminus S_i\}}{Max} \left\{ \sum_{j \in J_i} \hat{a}_{ij} y_j + (\Gamma_i - \lfloor \Gamma_i \rfloor) a_{it_i} y_{t_i} \right\} \leqslant b_i \quad \forall i, \\ & -y_j \leqslant x_j \leqslant y_j \quad \forall j, \\ & l_j \leqslant x_j \leqslant u_j \quad \forall j, \\ & y_j \geqslant 0 \quad \forall j. \end{aligned} \tag{23}$$

Finally, they prove that model (21) has a robust counterpart as follows:

$$\text{Max}\quad z = \mathbf{cx},$$

$$\text{s.t.}:\quad \sum_j a_{ij} x_j + z_i \Gamma_i + \sum_{j \in J_i} p_{ij} \leqslant b_i \quad \forall i, \tag{24}$$

$$z_i + p_{ij} \geqslant \hat{a}_{ij} y_j \quad \forall i, j \in J_i, \tag{25}$$

$$-y_j \leqslant x_j \leqslant y_j \quad \forall j, \tag{26}$$

$$l_j \leqslant x_j \leqslant u_j \quad \forall j, \tag{27}$$

$$p_{ij} \geqslant 0 \quad \forall i, j \in J_i, \tag{28}$$

$$y_j \geqslant 0 \quad \forall j, \tag{29}$$

$$z_i \geqslant 0 \quad \forall i. \tag{30}$$

Now, consider the following linear optimization model:

$$\begin{aligned} Min\quad & z = \sum_j c_j x_j, \\ & \sum_j a_{ij} x_j \leqslant \tilde{b}_i \quad \forall i, \\ & x_j \geqslant 0 \quad \forall j, \end{aligned} \tag{31}$$

in which c_{ij} and a_{ij} are deterministic and $\tilde{b}_i$ is uncertain. Moreover, each uncertain right-hand side ($\tilde{b}_i$) is the summation of some uncertain parameters. That is,

$$\tilde{b}_i = \sum_{s=1}^{\tau_i} \tilde{b}_{is}, \tag{32}$$

where τ_i is the number of uncertain parameters in ith constraint. Moreover, each uncertain parameter ($\tilde{b}_{is}$) takes a value according to a symmetric distribution with mean equals to the nominal value b_{is} in the interval $[b_{is} - \hat{b}_{is}, b_{is} + \hat{b}_{is}]$. In addition, parameter Γ_i is defined for every constraint I similar to one defined by Bertsimas and Sim (2004). This parameter takes a value in the interval $[0, |\tau_i|]$, and adjusts the level of conservatism of the solution acquired by our proposed method. As mentioned earlier, according to this method the acquired solution is feasible when up to $\lfloor \Gamma_i \rfloor$ parameters of ith right-hand side are permitted to change and one parameter of ith right-hand side ($\tilde{b}_{it_i}$) changes by $((\Gamma_i - \lfloor \Gamma_i \rfloor)\hat{b}_{it_i})$. Now according to Eq. (32), the constraints of model (31) can be rewritten as,

$$\sum_j a_{ij}x_j \leqslant \tilde{b}_i = \sum_{s=1}^{\tau_i} \tilde{b}_{is}. \tag{33}$$

Obviously, we should also define a protective function $\beta(\tau_i, \Gamma_i)$ to achieve a feasible solution when up to $\lfloor \Gamma_i \rfloor$ right-hand sides change and one parameter $\tilde{b}_{it_i}$ changes by $(\Gamma_i - \lfloor \Gamma_i \rfloor)\hat{b}_{it_i}$. Therefore, each right-hand side could be written according to the nominal value and protective function as follows:

$$\tilde{b}_i = \sum_{s=1}^{\tau_i} \tilde{b}_{is} = \sum_{s=1}^{\tau_i} b_{is} - \beta(\tau_i, \Gamma_i). \tag{34}$$

To achieve a feasible solution in the worst condition of allowed changes, $\beta(\tau_i, \Gamma_i)$ should be defined as follows:

$$\beta(\tau_i, \Gamma_i) = \underset{\{S_i \cup t_i | S_i \subseteq \tau_i, |S_i| = \lfloor \Gamma_i \rfloor, t_i \in \tau_i \setminus S_i\}}{Max} \left\{ \sum_{s \in \tau_i} \hat{b}_{is} + (\Gamma_i - \lfloor \Gamma_i \rfloor)\hat{b}_{it_i} \right\}. \tag{35}$$

Now, according to Eq. (34), Eq. (33) can be rewritten as follows:

$$\sum_j a_{ij}x_j \leqslant \sum_{s=1}^{\tau_i} b_{is} - \beta(\tau_i, \Gamma_i) \Rightarrow \sum_j a_{ij}x_j + \beta(\tau_i, \Gamma_i) \leqslant \sum_{s=1}^{\tau_i} b_{is} = b_i. \tag{36}$$

Thus, the related non-linear robust optimization model is,

$$\begin{aligned} &\text{Max} \quad w = \sum_j - c_j x_i, \\ &\sum_j a_{ij}x_{ij} + \underset{\{S_i \cup t_i | S_i \subseteq \tau_i, |S_i| = \lfloor \Gamma_i \rfloor, t_i \in \tau_i \setminus S_i\}}{Max} \left\{ \sum_{s \in \tau_i} \hat{b}_{is} + (\Gamma_i - \lfloor \Gamma_i \rfloor)\hat{b}_{it_i} \right\} \leqslant b_i \quad \forall i, \\ &x_j \geqslant 0 \quad \forall i, j. \end{aligned} \tag{37}$$

Comparing the acquired model to the initial model (22) demonstrates that the current model is a simplified version of the former in which $u_j = \infty$, $l_j = 0$ and $\hat{a}_{ij} = \hat{b}_{ij}$. Moreover, since the new protective function (Eq. (35)) does not include any decision variable, variables y in constraint (25) equal one and constraint (26) can be removed. In Appendix A (Proposition 1) it is proven that the robust counterpart of model (31) is as follows:

$$\begin{aligned} &\text{Max} \quad w = \sum_j - c_j x_j \text{ or Min} \quad z = \sum_j c_j x_j, \\ &\text{s.t.}: \quad \sum_j a_{ij}x_j + z_i\Gamma_i + \sum_{s \in \tau_i} p_{is} \leqslant b_i \quad \forall i, \\ &z_i + p_{is} \geqslant \hat{b}_{is} \quad \forall i, s \in \tau_i, \\ &x_j \geqslant 0 \quad \forall j, \\ &p_{is} \geqslant 0 \quad \forall i, s \in \tau_i, \\ &z_i \geqslant 0 \quad \forall i. \end{aligned} \tag{38}$$

Moreover, it can be shown that the acquired probability bounds of constraint violation by Bertsimas and Sim (2004) also hold for the above counterpart of the model (Proposition 2). In other words, one could prove that if more than $\lfloor \Gamma_i \rfloor$ parameters of ith right-hand side change; the probability of the ith constraint violation is at most $B(n, \Gamma_i)$ (Proposition 3). Where,

$$B(n, \Gamma_i) = \frac{1}{2^n} \left\{ (1-\mu)\binom{n}{\lfloor v \rfloor} + \sum_{l=\lfloor v \rfloor + 1}^{n} \binom{n}{l} \right\}, \tag{39}$$

in which, $n = |\tau_i| v = (\Gamma_i + n)/2$ and $\mu = v - \lfloor v \rfloor$.

To prove this claim, we use Proposition 2 and Theorem 3 proven by Bertsimas and Sim (2004). Note that, in the new proof, parameters η_{ij}, r^* and γ_{ij} are respectively replaced by η_{is}, new r^* and γ_{is} defined as follows,

$$\eta_{is} = \frac{b_{is} - \tilde{b}_{is}}{\hat{b}_{is}}, \tag{40}$$

$$r^* = \arg\max_{s \in \tau_i} \{b_{is}\}, \tag{41}$$

$$\gamma_{is} = \begin{cases} 1 & \text{if } s \in S_i^* \\ \hat{b}_{is}/\hat{b}_{ir^*} & \text{if } s \notin S_i^* \end{cases}. \tag{42}$$

These proofs are presented in Appendix A. The presented proofs demonstrate that model (38) results in feasible solution when utmost $\lfloor \Gamma_i \rfloor$ right-hand sides are allowed to change and one parameter $\tilde{b}_{it_i}$ changes by $(\Gamma_i - \lfloor \Gamma_i \rfloor)\hat{b}_{it_i}$. In addition, when the number of changed parameters is more than the number of parameters permitted to change; the probability of ith constraint's violation is less than or equal to $B(n, \Gamma_i)$.

4. Robust formulation of SMLM

The customized robust approach was proposed for a stochastic model in which all constraints are "less than or equal to" inequalities. Therefore, to use this approach to present the robust counterpart, all constraints of the stochastic model must be of the same type. As a result, the constraints of SMLM containing uncertain parameters may need to be converted. Since all uncertain constraints except constraints (4) and (5) are of a "less than or equal to" form, the conversion remains limited to constraints (4) and (5). For constraint (4),expression $Surp_{amt}$ is removed, and it is rewritten as

$$\sum_{v=1}^{V}\sum_{l=1}^{L}\sum_{s=1}^{t}\sum_{p=1}^{P} X_{amps}^{lv} - \sum_{v=1}^{V}\sum_{l=1}^{L}\sum_{s=1}^{t}\sum_{o=1}^{O} X_{aoms}^{lv}.\delta_{somt}^{v} - dev_{amt} \leqslant -\sum_{s=1}^{t} \tilde{d}_{ams}. \tag{43}$$

In addition, since the second objective function minimizes the amount of unsatisfied demand, the equality sign could be easily replaced by a "less than or equal to" sign in constraint (5). So, it can be written as follows:

$$\sum_{v=1}^{V}\sum_{s=1}^{t}\sum_{q=1}^{Q}\sum_{p=1}^{P} Y_{hqps}^{mv} - \sum_{v=1}^{V}\sum_{s=1}^{t}\sum_{q=1}^{Q}\sum_{o=1}^{O} Y_{hoqs}^{mv}.\delta_{soqt}^{v} - dew_{hmt} \leqslant \sum_{s=1}^{t} \tilde{w}_{hms}. \tag{44}$$

Now, to determine the robust counterpart of SMLM, named RMLM for Robust Model for Logistics Management, the following additional variables are defined.

Γ_{am}^{1t}: Number of uncertain parameter in constraint (43) until period t.

α_{am}^{1t} and β_{ams}^{1t}: Dual variables of the linear equivalent of protective function of constraint (43).

Likewise, Γ_{hm}^{2t} is defined for constraints (7) and (44), Γ_{al}^{3t} and Γ_{hq}^{4t} are respectively defined for constraints (6) and (19). Moreover, α_{hm}^{2t}, β_{hms}^{2t} are defined as the dual variables of the protective functions of constraints (4) and (44). In addition, α_{al}^{3t}, β_{als}^{3t}, α_{hq}^{4t} and β_{hqs}^{4t} are defined as the dual variables of protective function of constraints (6) and (19). The RMLM model can therefore be written as follows:

$$\text{Min } f_1 = \sum_{h=1}^{H}\sum_{m=1}^{M}\sum_{t=1}^{T} P'_h \cdot dew_{hmt}, \tag{45}$$

$$\text{Min } f_2 = \sum_{a=1}^{A}\sum_{m=1}^{M}\sum_{t=1}^{T} P_a \cdot dev_{amt}, \tag{46}$$

$$\text{Min } f_3 = \sum_{o=1}^{O}\sum_{p=1}^{P}\sum_{t=1}^{T} Z_{opt}, \tag{47}$$

$$\text{S.t.}: \sum_{v=1}^{V}\sum_{l=1}^{L}\sum_{s=1}^{t}\sum_{p=1}^{P} X_{amps}^{lv} - \sum_{v=1}^{V}\sum_{l=1}^{L}\sum_{s=1}^{t}\sum_{o=1}^{O} X_{aoms}^{lv} \cdot \delta_{somt}^{v} - dev_{amt} + \sum_{s\in\tau_1 \,\&\, s\leqslant t} \beta_{ams}^{1t} + \alpha_{am}^{1t}\Gamma_{am}^{1t} \leqslant -\sum_{s=1}^{t} d_{ams} \quad \forall m \in DN,\ a \in CS,\ t \in T, \tag{48}$$

$$\sum_{v=1}^{V}\sum_{s=1}^{t}\sum_{q=1}^{Q}\sum_{p=1}^{P} Y_{hqps}^{mv} - \sum_{v=1}^{V}\sum_{s=1}^{t}\sum_{q=1}^{Q}\sum_{o=1}^{O} Y_{hoqs}^{mv} \cdot \delta_{soqt}^{v} - dew_{hmt} + \sum_{s\in\tau_2 \,\&\, s\leqslant t} \beta_{hms}^{2t} + \alpha_{hm}^{2t}\Gamma_{hm}^{2t} \leqslant -\sum_{s=1}^{t} w_{hms} \quad \forall m \in DN,\ h \in IS,\ t \in T, \tag{49}$$

$$\sum_{v=1}^{V}\sum_{s=1}^{t}\sum_{p=1}^{P} X_{alps}^{lv} + \sum_{s\in\tau_3 \,\&\, s\leqslant t} \beta_{als}^{3t} + \alpha_{al}^{3t}\Gamma_{al}^{3t} \leqslant \sum_{s=1}^{t} \sup_{als} \quad \forall a \in CS,\ l \in SN,\ t \in T, \tag{50}$$

$$\sum_{v=1}^{V}\sum_{s=1}^{t}\sum_{p=1}^{P} Y_{hmps}^{mv} \sum_{s\in\tau_2 \,\&\, s\leqslant t} \beta_{hms}^{2t} + \alpha_{hm}^{2t}\Gamma_{hm}^{2t} \leqslant \sum_{s=1}^{t} w_{hms} \quad \forall m \in DN,\ h \in IS,\ t \in T, \tag{51}$$

$$\sum_{v=1}^{V}\sum_{o=1}^{O}\sum_{s=1}^{t} X_{aoks}^{lv} \cdot \delta_{sokt}^{v} - \sum_{v=1}^{V}\sum_{p=1}^{P}\sum_{s=1}^{t} X_{akps}^{lv} = 0 \quad \forall a \in CS,\ k \in KN,\ l \in CS,\ t \in T,\ k \neq l, \tag{52}$$

$$\sum_{v=1}^{V}\sum_{o=1}^{O}\sum_{s=1}^{t} Y_{hons}^{mv} \cdot \delta_{sont}^{v} - \sum_{v=1}^{V}\sum_{p=1}^{P}\sum_{s=1}^{t} Y_{hnps}^{mv} = 0 \quad \forall h \in IS,\ n \in NN,\ m \in DN,\ t \in T,\ n \neq m, \tag{53}$$

$$\sum_{l=1}^{L}\sum_{o=1}^{O}\sum_{p=1}^{P} X_{aopt}^{lv} \leqslant M_{Big} \cdot ac_a^v \quad \forall a \in CS,\ v \in VS,\ t \in T, \tag{54}$$

$$\sum_{m=1}^{M}\sum_{o=1}^{O}\sum_{p=1}^{P} Y_{hopt}^{mv} \leqslant M_{Big} \cdot aw_h^v \quad \forall h \in IS,\ v \in VS,\ t \in T, \tag{55}$$

$$\sum_{o=1}^{O}\sum_{l=1}^{L}\sum_{s=1}^{t} X_{aois}^{lv}\left(\delta_{soit}^{v} - \delta_{soi(t-1)}^{v}\right) - \sum_{l=1}^{L}\sum_{p=1}^{P} X_{aipt}^{lv} = TF_{ait}^{v} - TT_{ait}^{v} \quad \forall a \in CS,\ i \in IN,\ t \in T,\ v \in VS, \tag{56}$$

$$\sum_{o=1}^{O}\sum_{m=1}^{M}\sum_{s=1}^{t} Y_{hois}^{mv}\left(\delta_{soit}^{v} - \delta_{soi(t-1)}^{v}\right) - \sum_{m=1}^{M}\sum_{p=1}^{P} Y_{hipt}^{mv} = TFD_{hit}^{v} - TTD_{hit}^{v} \quad \forall h \in IS,\ i \in IN,\ t \in T,\ v \in VS, \tag{57}$$

$$\sum_{l=1}^{L}\sum_{a=1}^{A} X_{aopt}^{lv} \cdot c_a \leqslant Z_{opt}^{v} \cdot cc^{v} \quad \forall o \in N,\ p \in N,\ t \in T,\ v \in VS, \tag{58}$$

$$\sum_{l=1}^{L}\sum_{a=1}^{A} X_{aopt}^{lv} \cdot w_a \leqslant Z_{opt}^{v} \cdot cw^{v} \quad \forall o \in N,\ p \in N,\ t \in T,\ v \in VS, \tag{59}$$

$$\sum_{m=1}^{M}\sum_{h=1}^{H} Y_{hopt}^{mv} \leqslant Z_{opt}^{v} \cdot dv^{v} \quad \forall o \in N,\ p \in N,\ t \in T,\ v \in VS, \tag{60}$$

$$Z_{opt}^{v} \leqslant M_{Big} \cdot t_{op}^{v} \quad \forall o \in N,\ p \in N,\ t \in T,\ v \in VS, \tag{61}$$

$$\sum_{o=1}^{O}\sum_{s=1}^{S} Z_{ops}^{v} \cdot \delta_{sopt}^{v} + \sum_{s=1}^{t} av_{ps}^{v} = avv_{pt}^{v} + \sum_{o=1}^{O}\sum_{s=1}^{t} Z_{pos}^{v} \quad \forall p \in N,\ t \in T,\ v \in VS, \tag{62}$$

$$\sum_{v=1}^{V}\sum_{m=1}^{M}\sum_{o=1}^{O}\sum_{s=1}^{t} Y_{hoqs}^{mv} \cdot \delta_{soqt}^{v} - \sum_{v=1}^{V}\sum_{m=1}^{M}\sum_{p=1}^{P}\sum_{s=1}^{S} Y_{hqps}^{mv} + \sum_{s \in \tau_4\ \&\ s \leqslant t} \beta_{hqs}^{4t} + \alpha_{hq}^{4t}\Gamma_{hq}^{4t} \leqslant \sum_{s=1}^{t} wsp_{hqs} \quad \forall h \in IS,\ q \in HN,\ v \in VS, \tag{63}$$

$$\beta_{ams}^{1t} + \alpha_{am}^{1t}\Gamma_{am}^{1t} \geqslant \hat{d}_{ams} \quad \forall a \in CS,\ m \in DN,\ t \in T,\ s \in \tau_1\ \&\ s \leqslant t, \tag{64}$$

$$\beta_{hms}^{2t} + \alpha_{hm}^{2t}\Gamma_{hm}^{2t} \geqslant \hat{w}_{hms} \quad \forall h \in IS,\ m \in DN,\ t \in T,\ s \in \tau_2\ \&\ s \leqslant t, \tag{65}$$

$$\beta_{als}^{3t} + \alpha_{al}^{3t}\Gamma_{al}^{3t} \geqslant S\hat{u}p_{als} \quad \forall a \in CS,\ l \in SN,\ t \in T,\ s \in \tau_3\ \&\ s \leqslant t, \tag{66}$$

$$\beta_{hqs}^{4t} + \alpha_{hq}^{4t}\Gamma_{hq}^{4t} \geqslant w\hat{s}p_{hqs} \quad \forall h \in IS,\ q \in HN,\ t \in T,\ s \in \tau_4\ \&\ s \leqslant t, \tag{67}$$

$$\begin{aligned} & Y \geqslant 0\ \&\ Integer, \quad X \geqslant 0\ \&\ Integer, \quad Z \geqslant 0\ \&\ Integer, \\ & dev \geqslant 0\ \&\ Integer, \quad dew \geqslant 0\ \&\ Integer, \quad avv \geqslant 0\ \&\ Integer, \\ & \beta \geqslant 0, \quad \alpha \geqslant 0, \end{aligned} \tag{68}$$

where constraints (48) and (64) are the robust counterparts of constraint (43), and constraints (50) and (66) are the robust counterparts of constraint (6) of SMLM model. Moreover, the robust counterpart of constraint (44) includes constraints (49) and (65). Finally, constraints (51), (65), (63), (67) are respectively robust counterparts of constrains (7) and (19).

5. Solution methodology for the RMLM model

Following the hierarchical objective functions of the RMLM model, it can be reformulated as a Structured RMLM (SRMLM) as follows:

$$\begin{aligned} \text{Min} \quad & f_3(Z), \\ & \text{s.t.}: \\ & \left\{ \begin{array}{l} \text{Min } f_2(X), \\ \text{s.t.}: \\ \left\{ \begin{array}{l} g_i^1(X) \leqslant b_i^1, \\ g_i^2(X,Z) \leqslant b_i^2, \\ g_i'(\alpha,\beta) \leqslant b_i', \\ Z \in \left\{ \begin{array}{l} \text{Min } f_1(Y), \\ \text{s.t.}: \\ g_i^3(Y) \leqslant b_i^3, \\ g_i^4(Y,Z) \leqslant b_i^4, \\ g_i^5(Z) \leqslant b_i^5, \qquad\qquad (69-\text{I}) \\ g_i''(\alpha,\beta) \leqslant b_{,i}'' \\ Y,Z \in Int^+, \alpha^2, \beta^2, \alpha^4, \beta^4 \geqslant 0, \end{array} \right. \\ X \in Int^+, \alpha^1, \beta^1, \alpha^3, \beta^3 \geqslant 0, \end{array} \right. \end{array} \right. \end{aligned} \tag{69}$$

where $f_1(Y), f_2(X)$ and $f_3(Z)$ are respectively the first, the second and the third objective function. Furthermore, $g_i^1(X)$ denotes the set of all constraints of RMLM being function of X variable, and $g_i^2(X,Z)$ denotes the set of all constraints of RMLM being function of X and Z variables, and so on. Now, to solve the SRMLM model, three linear optimization model P_1, P_2 and P_3 are defined as follows,

Model P_1

$$\begin{aligned}
\text{Min}\quad & f_1(Y),\\
\text{s.t.}:\quad & g_i^3(Y) \leqslant b_i^3,\\
& g_i^4(Y,Z) \leqslant b_i^4,\\
& g_i^5(Z) \leqslant b_i^5,\\
& g_i''(\alpha,\beta) \leqslant b_i'',\\
& Y,Z \in Int^+, \alpha^2, \beta^2, \alpha^4, \beta^4 \geqslant 0.
\end{aligned} \tag{70}$$

Model P_2

$$\begin{aligned}
\text{Min}\quad & f_2(X),\\
\text{s.t.}:\quad & f_1(Y) \leqslant f_1'^{*}, \quad \text{(I)}\\
& g_i^1(X) \leqslant b_i^1,\\
& g_i^2(X,Z) \leqslant b_i^2, \quad \text{(II)}\\
& g_i'(\alpha,\beta) \leqslant b_i',\\
& g_i^3(Y) \leqslant b_i^3,\\
& g_i^4(Y,Z) \leqslant b_i^4, \quad \text{(III)}\\
& g_i^5(Z) \leqslant b_i^5, \quad \text{(IV)}\\
& g_i''(\alpha,\beta) \leqslant b_i'',\\
& X,Y,Z \in Int^+, \alpha,\beta \geqslant 0.
\end{aligned} \tag{71}$$

Model P_3

$$\begin{aligned}
\text{Min}\quad & f_3(Z),\\
\text{s.t.}:\quad & g_i^2(x_2^*,Z) \leqslant b_i^2,\\
& g_i^4(y_2^*,Z) \leqslant b_i^4,\\
& g_i^5(Z) \leqslant b_i^5,\\
& Z \in Int^+,
\end{aligned} \tag{72}$$

where $f_1'^{*}$, x_2^* and y_2^* are the optimal objective value of model P_1 and the optimal values of variables X and Y in model P_2. Note that, since objectives are hierarchical, they are handled lexicographically. In other words, the first objective function is optimized without taking the other objective functions into account in the first step. In addition, the first step does not consider the constraints of the X variable. Next, the first objective is fixed to its optimal value (which was obtained in the previous step, and is named f_1^*), and the second objective is minimized according to the X and Y constraints. Because the value of the first objective function is considered as a constraint in this step, the approach guarantees that the optimization of the first objective is prioritized over the second one. Finally, by fixing the values of the X and Y variables, the proposed methodology tries to minimize the number of vehicles used in the disaster response phase. According to the defined models, the optimal solution of SRMLM could be achieved by the methodology depicted in Fig. 1. For the ease of reference within the paper, this methodology is named SMSRM (Solution Methodology of the Structured Robust Model).

Proposition SMSRM methodology achieves the optimal solution if SRMLM has a single optimal solution. Otherwise, this methodology attains one of the optimal solutions.

Proof. Let us assume that (Y_1^*, Z_1^*), (X_2^*, Y_2^*, Z_2^*), (Z_3^*) and (X^*, Y^*, Z^*) are respectively the optimal solutions of models P_1, P_2, P_3 and SRMLM. In addition, it is assumed that $f_1'^{*}, f_2'^{*}, f_3'^{*}$ are the optimal objective values of models P_1, P_2, P_3, and (f_1^*, f_2^*, f_3^*) are the optimal value of objectives in SRMLM. Moreover, FA_1, FA_2, FA_3 and FA, respectively, denote the feasible area of models P_1, P_2, P_3 and SRMLM. Furthermore, since all constraints of model P_1 exist in model P_2, FA_2 is a subset of FA_1 ($FA_2 \subseteq FA_1$). A similar argument holds for SRMLM. Therefore, FA_2 is also a subset of FA ($FA_2 \subseteq FA$). Now, to prove the proposition, two states – namely a single optimal solution and multiple optimal solutions-are considered separately.

State I. The SRMLM model has a single optimal solution.

f_1 is the first hierarchical objective function of the SRMLM. Therefore, to achieve its optimal value, only constraints (69-I) must be considered. In fact, this subset of SRMLM is model P_1. Thus, f_1^* is equal to $f_1'^*$. By replacing the value of $f_1'^*$ in model P_2, it could be claimed that $Y_2^* = Y^*$. To prove this claim, consider constraint (71-I) in model P_2. According to this constraint, $f_1(Y_2^*)$ could not be more than $f_1'^* \left(f_1(Y_2^*) \ngtr f_1'^*\right)$. Furthermore, since

$$\begin{cases} Y_2^*, Z_2^* \in FA_2 \\ FA_2 \subseteq FA_1 \end{cases} \Rightarrow Y_2^*, Z_2^* \in FA_1. \tag{73}$$

Now, since $f_1'^*$ is the optimal value of the objective function, $f_1(Y_2^*) \nless f_1'^*$. So,

$$\begin{cases} f_1(Y_2^*) \nless f_1'^* \\ f_1(Y_2^*) \ngtr f_1'^* \end{cases} \Rightarrow f_1(Y_2^*) = f_1'^* = f_1^* = f_1(Y_2). \tag{74}$$

Finally, since it is presumed that the SRMLM has a single optimal solution in this state, the equality of these two objective functions guarantees the equality of Y_2^* andY^*. Furthermore, the equality of X_2^* and X^* is proven by contradiction. Suppose that $X_2^* \neq X^*$. Since the SRMLM model has a single optimal solution (according to State I), the objective values of these two solutions could not be equal $(f_2(X_2^*) \neq f_2(X^*))$. So, one of the following cases could take place.

Case A. $f_2(X_2^*) < f_2(X^*)$,
Since (X_2^*, Y_2^*, Z_2^*) is the optimal solution of model P_2, it is a feasible solution for this model. On the other hand, $Y_2^* = Y^*$. So, (X_2^*, Y^*, Z_2^*) is also a feasible solution for model P_2. Therefore,

$$\begin{cases} (X_2^*, Y^*, Z_2^*) \in FA_2 \\ FA_2 \subseteq FA \end{cases} \Rightarrow (X_2^*, Y^*, Z_2^*) \in FA. \tag{75}$$

Eq. (75) demonstrates that (X_2^*, Y^*, Z_2^*) is a feasible solution for the SRMLM. Moreover, as mentioned earlier, the objectives of SRMLM are hierarchically structured. In other words, the optimization of the first objective is more desirable than optimization of the second or third objectives. Now, if we assume that $f_1(Y_2^*) = f_1(Y^*)$ and $f_2(X_2^*) < f_2(X^*), X^*$ could not be the optimal solution of SRMLM model. This conclusion contradicts with the initial assumption on X^*. Thus,

$$f_2(X_2^*) \nless f_2(X^*). \tag{76}$$

Case B. $f_2(X_2^*) > f_2(X^*)$,
In this case, since $(X^*, Y^*, Z^*) \in FA$; all constraints of SRMLM are satisfied. In addition, since all constraint of model P_2 except (71-I) exist in SRMLM, and $f_1(Y^*) = f_1^* = f_1'^*$, all constraints of model P_2 are also satisfied. Therefore, (X^*, Y^*, Z^*) is a feasible solution for model P_2. Now, because $f_1(Y^*) = f_1'^*$, and $f_2(X^*) < f_2(X_2^*)$, X_2^* could not be the optimal solution of model P_2. This conclusion also contradicts with our assumption about the optimality of X_2^*. So,

$$f_2(X_2^*) \ngtr f_2(X^*). \tag{77}$$

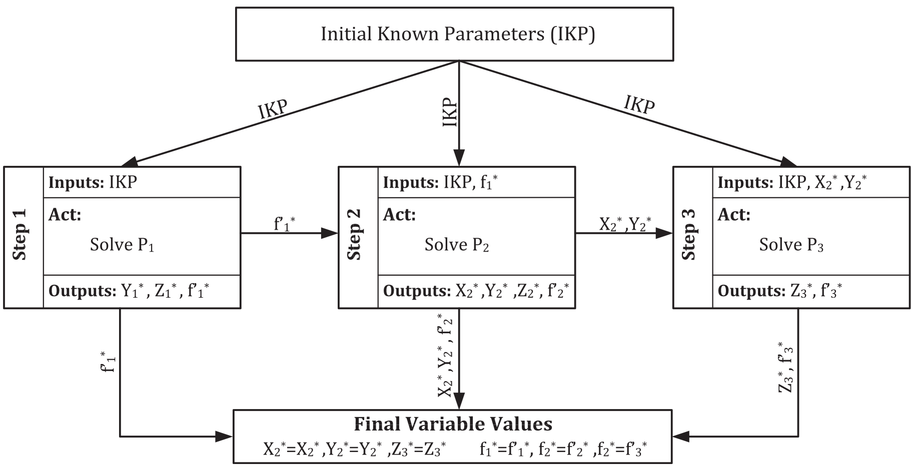

Fig. 1. SMSRM methodology for solving SRMLM model.

Thus, Eqs. (76) and (77) show that $f_2(X_2^*) = f_2(X^*)$. Now, since the SRMLM model has a single optimal solution in this state, it is concluded that $X_2^* = X^*$.

Finally, to prove the equality of Z_3^* and Z^*, we also utilize the contradiction method. Suppose that $Z_3^* \neq Z^*$. According to the single optimal solution of SRMLM in State I, $f_3(Z_3^*) \neq f_3(Z^*)$. Therefore, one of the following cases is allowed to occur.

Case C. $f_3(Z_3^*) < f_3(Z^*)$,

In this case, since

$$\begin{cases} X_2^* = X^*, \\ Y_2^* = Y^*, \\ (X_2^*, Y_2^*, Z_3^*) \in FA_3, \end{cases} \Rightarrow (X^*, Y^*, Z_3^*) \in FA_3. \tag{78}$$

Moreover, since all constraints of model P_3 exist in model P_2, it can be concluded that

$$(X_2^*, Y_2^*, Z_3^*) \in FA_2. \tag{79}$$

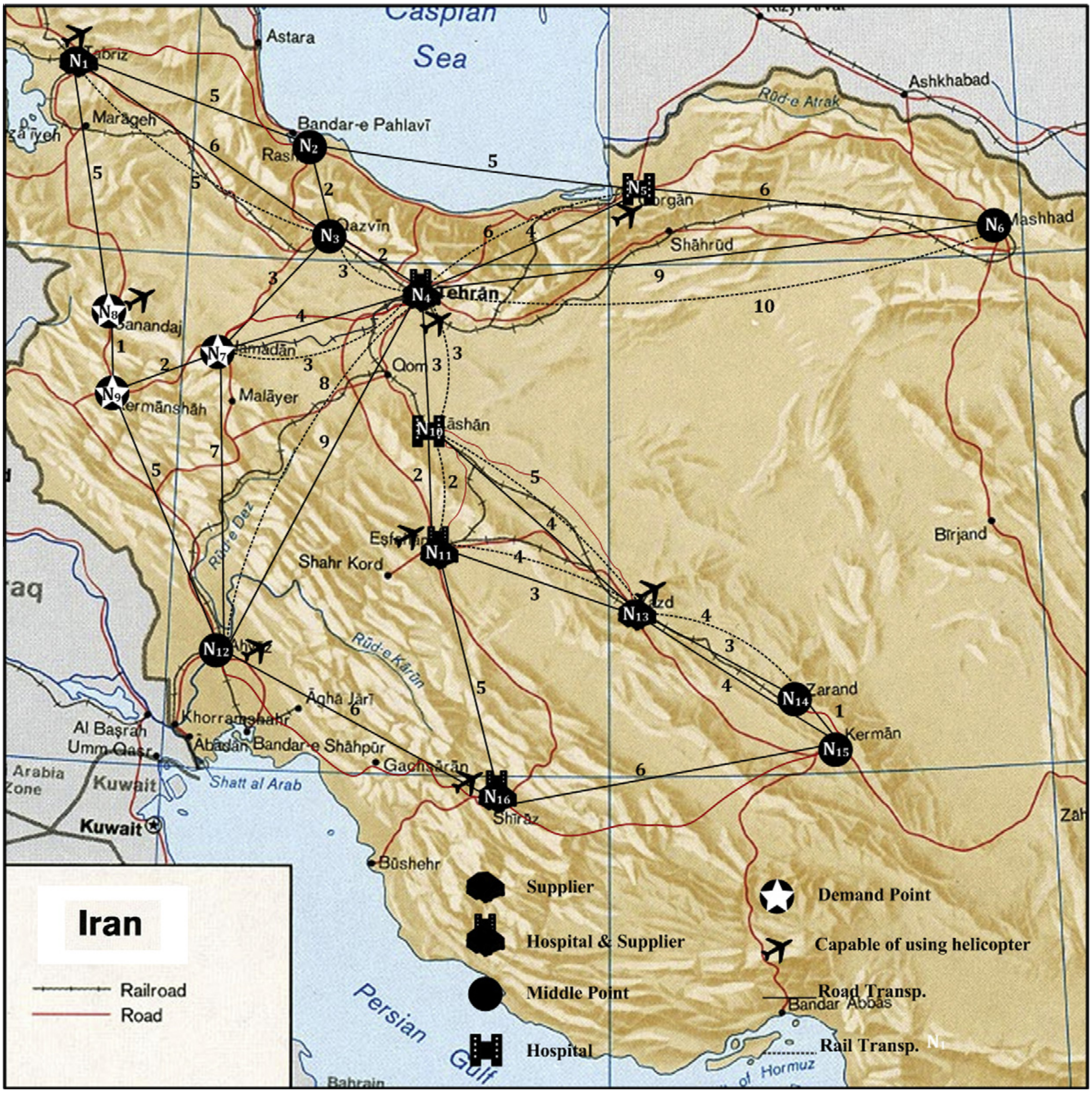

Fig. 2. The considered network in the illustrative example.

Table 3
Commodity demand and injured people (units) at nodes.

Time	4			7			9			14			17			24		
Node	N_7	N_8	N_9	N_7	N_8	N_9	N_7	N_8	N_9	N_7	N_8	N_9	N_7	N_8	N_9	N_7	N_8	N_9
A_1	60	80	24	54	96	30	80	100	36	100	75	48	80	70	40	68	60	38
$\hat{A}_1$	6	8	2	5	10	3	8	10	4	10	8	5	8	7	4	7	6	4
A_2	50	30	20	44	40	35	60	50	30	40	45	40	30	50	26	20	35	16
$\widehat{A}_2$	5	3	2	5	4	4	6	5	3	4	5	4	3	5	3	2	4	2
H_1	20	32	11	14	20	7	10	15	5	–	13	–	7	7	6	3	9	5
$\widehat{H}_1$	2	3	1	1	2	1	1	2	1	–	1	–	1	1	1	1	1	1
H_2	28	32	30	22	25	36	17	23	27	–	–	21	9	15	18	6	12	9
$\widehat{H}_2$	3	3	3	2	3	4	2	2	3	–	–	2	1	2	2	1	1	1

Table 4
Commodity supply (units).

Time	1		3		8		11		15		19	
Com.	A_1	$\widehat{A}_1$	A_1	$\widehat{A}_1$	A_1	$\widehat{A}_1$	A_1	$\widehat{A}_1$	A_1	$\widehat{A}_1$	A_1	$\widehat{A}_1$
N_1	30	2	20	1	0	0	80	4	0	0	0	0
N_4	20	1	0	0	80	4	40	2	50	3	100	5
N_{11}	40	3	75	3	0	0	100	5	0	0	0	0
N_{13}	30	2	0	0	50	3	0	0	80	4	0	0
N_{16}	25	2	0	0	90	4	60	3	40	2	30	2
Com.	A_2	$\widehat{A}_2$	A_2	$\widehat{A}_2$	A_2	$\widehat{A}_2$	A_2	$\widehat{A}_2$	A_2	$\widehat{A}_2$	A_2	$\widehat{A}_2$
N_1	40	2	0	0	30	2	0	0	0	0	20	1
N_4	30	2	20	1	0	0	0	0	70	4	45	2
N_{11}	50	3	0	0	55	3	50	2	0	0	0	0
N_{13}	10	1	25	2	0	0	20	1	50	3	0	0
N_{16}	20	1	30	2	0	0	0	0	0	0	60	3

Table 5
Hospitals' capacity levels (persons/period).

Time	1		3		8		11		15		19	
Com.	H_1	$\widehat{H}_1$	H_1	$\widehat{H}_1$	H_1	$\widehat{H}_1$	H_1	$\widehat{H}_1$	H_1	$\widehat{H}_1$	H_1	$\widehat{H}_1$
N_4	40	2	–	–	30	2	–	–	–	–	30	2
N_5	20	1	–	–	–	–	15	1	–	–	–	–
N_{10}	15	1	–	–	15	1	–	–	–	–	–	–
N_{11}	25	1	20	1	–	–	–	–	10	1	–	–
N_{16}	30	2	–	–	–	–	15	1	–	–	–	–
Com.	H_2	$\widehat{H}_2$	H_2	$\widehat{H}_2$	H_2	$\widehat{H}_2$	H_2	$\widehat{H}_2$	H_2	$\widehat{H}_2$	H_2	$\widehat{H}_2$
N_4	60	3	–	–	15	1	30	2	30	2	–	–
N_5	30	2	–	–	20	1	–	–	10	1	–	–
N_{10}	20	1	–	–	–	–	20	1	–	–	20	1
N_{11}	40	2	–	–	–	–	20	1	–	–	–	–
N_{16}	30	2	–	–	25	1	–	–	30	1	30	2

Table 6
Vehicles capacity for commodity and injured people (units).

DV(v)	CC(**v**)	CW(v)	Vehicle
0	50	45	V_1
0	250	200	V_2
5	30	25	V_3
2	0	0	V_4

Therefore, according to Eq. (79) and equality of X_2^*, X^* and Y_2^*, Y^*, it is deduced that $(X^*, Y^*, Z_3^*) \in FA_2$. Moreover, since $FA_2 \subseteq FA$, $(X^*, Y^*, Z_3^*) \in FA$. Now, because $f_3(Z_3^*) < f_3(Z^*)$, Z^* cannot be the optimal solution of the SRMLM. This conclusion contradicts the assumption of optimality of Z^*. Thus,

$$f_3(Z_3^*) \not< f_3(Z^*). \tag{80}$$

Case D. $f_3(Z_3^*) > f_3(Z^*)$,

In this case, since $(X^*, Y^*, Z^*) \in FA$, $X_2^* = X^*$, $Y_2^* = Y^*$ and $f_1(Y^*) = f_1^* = f_1'^*$; It is deduced that $(X^*, Y^*, Z^*) \in FA_2$ (Because all constraints of model P_2 except (71-I) exist in SRMLM, and constraint (71-I) is also satisfied because $f_1(Y^*) = f_1'^*$). Furthermore, according to the constraints (71-II), (71-III) and (71-IV) in model P_2

$$\begin{cases} g_i^2(X^*, Z^*) \leqslant b_i^2 \\ g_i^4(Y^*, Z^*) \leqslant b_i^4 \\ g_i^5(Z^*) \leqslant b_i^5 \end{cases} \Rightarrow (X^*, Y^*, Z^*) \in FA_3 \tag{81}$$

Moreover, since $X_2^* = X^*$ and $Y_2^* = Y^*$, it is concluded that $(X_2^*, Y_2^*, Z^*) \in FA_3$. Now, since $f_3(Z^*) < f_3(Z_3^*)$, Z_3^* could not be the optimal solution of model P_3. This result also contradicts with the assumption about the Z_3^* optimality. Thus,

$$f_3(Z_3^*) \not> f_3(Z^*). \tag{82}$$

According to the results acquired in two cases C and D, it is deduced that $f_3(Z_3^*) = f_3(Z^*)$, and because of the single optimal solution of SRMLM in this state, $Z_3^* = Z^*$.

State II. SRMLM model has multiple optimal solutions.

The proof for the state of multiple optimal solutions is similar to the one presented for State I. The only difference is that the equality of objectives does not necessarily result in the equality of variables. In other words, since SRMLM model has several optimal solutions, the acquired solution by SMSRM methodology may equal to or be different with the solution directly obtained in SRMLM model (X^*, Y^*, Z^*). However, these acquired variables, (X_2^*, Y_2^*, Z_3^*), surely result in the same objective values which are the most important components in the earthquake response. □

6. An illustrative example

To illustrate how the proposed model works and what results it produces, consider an earthquake in a network with 16 nodes, 91 transportation arcs and four types of vehicles including 30 trucks, four trains, 10 helicopters and 30 ambulances.

Table 7
Weight, volume and priority of commodities and priority of injuries.

Pr(h)	Wnd.	Pr(a)	C(a)	W(a)	Com.
0.65	H_1	0.3	2	1.5	A_1
0.35	H_2	0.7	1	1	A_2

Table 8
Number of vehicles added to nodes.

Time	1							2			3			5				**6**	
Node	N_1	N_4	N_5	N_{10}	N_{11}	N_{13}	N_{16}	N_{10}	N_{11}	N_{13}	N_4	N_5	N_{16}	N_1	N_4	N_{13}	N_{16}	N_4	N_{16}
V_1	4	7			5	3	5				2		2	1		1			
V_2	1	2						1		1									
V_3		3			2	1	2								1		1		
V_4		4	5	5	4		4		2			2			2			1	1

Table 9
Model complexity and computation times.

Model	#Constraints	#Variables	#Discrete variables	Solution time (s)
P_1	56,400	178,069	172,032	29.343
P_2	112,231	427,993	417,792	404.169
P_3	99,840	24,576	24,576	10.372

Table 10
Transportation schedule for the injured (value of *Y* variables).

Time	H_1	H_2	Vehicles		Dest.
			V_3	V_4	
N_4					
10	5	–	$V_3 = 1$		N_{16}
14	–	10	$V_3 = 2$		N_{16}
18	–	15	$V_3 = 3$		N_{16}
20	–	1	$V_3 = 1$		N_5
21	–	20	$V_3 = 4$		N_5
22	–	14	$V_3 = 3$		N_{16}
N_7					
3	2	–		$V_4 = 1$	N_4
5	17	3		$V_4 = 10$	N_4
6	8	–		$V_4 = 4$	N_4
7	5	7		$V_4 = 6$	N_4
8	2	–		$V_4 = 1$	N_4
9	5	9		$V_4 = 7$	N_4
10	2	6		$V_4 = 4$	N_4
11	–	12		$V_4 = 6$	N_4
12	–	2		$V_4 = 1$	N_4
13	–	14		$V_4 = 7$	N_4
14	7	1		$V_4 = 4$	N_4
15	–	7 + 4		$V_4 = 6$	N_4
17	–	10		$V_4 = 5$	N_4
19	2	14 + 5		$V_4 = 11$	N_4
N_8					
3	25	5	$V_3 = 6$		N_4
	5	–	$V_3 = 1$		N_{11}
4	–	5	$V_3 = 1$		N_{16}
5	6	19	$V_3 = 5$		N_{11}
6	4	1	$V_3 = 1$		N_4
	5	–	$V_3 = 1$		N_{16}
7	2	8	$V_3 = 2$		N_4
	8 + 6	1	$V_3 = 3$		N_{11}
8	3	2	$V_3 = 1$		N_4
	7 + 2	1	$V_3 = 2$		N_{11}
9	7 + 4	4	$V_3 = 3$		N_{11}
	–	15	$V_3 = 3$		N_4
10	11	8 + 1	$V_3 = 4$		N_{11}
11	1	14	$V_3 = 3$		N_{11}
12	19	1	$V_3 = 4$		N_4
	–	5	$V_3 = 1$		N_{16}
13	–	15	$V_3 = 3$		N_4
N_8					
13	–	3 + 1	$V_3 = 1$		N_{11}
14	6	5 + 4	$V_3 = 3$		N_4
15	5		$V_3 = 1$		N_4
	1	8	$V_3 = 2$		N_{11}
16	–	9 + 5	$V_3 = 3$		N_{11}
17	–	4	$V_3 = 1$		N_4
	–	10	$V_3 = 2$		N_{16}
18	–	5	$V_3 = 1$		N_{11}
19	4	1	$V_3 = 1$		N_4
	4	1	$V_3 = 1$		N_5
	–	5	$V_3 = 1$		N_{11}
20	3	2	$V_3 = 1$		N_4
	1	4	$V_3 = 1$		N_5
21	–	14	$V_3 = 3$		N_4
N_9					
4	6	–		$V_4 = 3$	N_8
6	6	–		$V_4 = 3$	N_8
7	5	3		$V_4 = 4$	N_8
8	4	4		$V_4 = 4$	N_8
9	–	8		$V_4 = 4$	N_8
10	–	14		$V_4 = 7$	N_8
11	–	6		$V_4 = 3$	N_8

Table 10 (*continued*)

Time	H_1	H_2	Vehicles		Dest.
			V_3	V_4	
12	–	18		$V_4 = 9$	N_8
13	–	4		$V_4 = 2$	N_8
14	6	8		$V_4 = 7$	N_8
	–	4		$V_4 = 2$	N_7
15	–	5		$V_4 = 3$	N_8
16	–	14		$V_4 = 7$	N_8
17	–	5		$V_4 = 3$	N_8
18	–	14		$V_4 = 7$	N_7
19	4	2		$V_4 = 3$	N_8
20	–	14		$V_4 = 7$	N_8
N_{11}					
5	5	–	$V_3 = 1$		N_{16}
16	–	10	$V_3 = 2$		N_{16}
17	–	6	$V_4 = 3$		N_{10}
18	–	2	$V_4 = 1$		N_{10}
19	–	4	$V_4 = 2$		N_{10}
20	–	4	$V_4 = 2$		N_{10}

Assume that the helicopters can carry both commodities and injured people, whereas trains and trucks can transport commodities, and ambulances are only authorized to carry injured people. In addition, there are five suppliers and five hospitals located at nodes N_1, N_4, N_{11}, N_{13}, N_{15} and nodes N_4, N_5, N_{10}, N_{11}, N_{16} respectively (see Fig. 2).

Now, suppose that an earthquake strikes this network and nodes N_7, N_8 and N_9 incur damage. There is no exact information available on the damage and therefore demand for two types of commodities and numbers of injured people (for two types of injuries) for the next 24 periods are presented in Table 3 by means of their averages and half ranges. Also, assume that there are two possible scenarios on the network availability: a base scenario and worst case scenario. The base scenario assumes that all roads of network are available and usable for response. However, the worst case scenario assumes that roads N_8–N_9 and N_7–N_9 are blocked and unusable for response activities. This example, without loss of generality, assumes that the commodities are measured by the number of items (and not e.g. in kg or liter). Furthermore, the uncertain available commodities at supply nodes and hospitals' capacities are presented in a similar fashion in Tables 4 and 5.

Tables 6–8 offer more information on the commodity and injury types, their priorities, vehicle properties and number of vehicles added to various nodes during the planning horizon. Moreover, because the number of uncertain parameters such as commodity needs, number of injured persons, and hospital capacities in the consecutive periods change over time, the number of allowed parameters for change is chosen variable, and assumed to be equal to half of all uncertain parameters, i.e. $\Gamma_i^t = 0.5\tau_i^t$.

As mentioned earlier, the proposed SMSRM methodology is used to hierarchically minimize the number of unserved injured people, the amount of unsatisfied demand, and the number of used vehicles in a lexicographic fashion.

The three corresponding models P_1, P_2 and P_3 are modeled in GAMS Rev 232 and solved by CPLEX 12.1 on a notebook computer with a Pentium (R) 2.0 MHz CPU and 4 GB DDR3 RAM. More information on the model properties and their solution times are reported in Table 9.

The acquired schedule for injured people transportation, commodity carrying and vehicle movement are depicted in Tables 10–12.

Table 12 describes the vehicles' movement schedule by means of three types of numbers. The bold numbers denote the vehicles shipping the commodity; the italic and underlined numbers refer to the vehicles transporting injured people, and the regular or plain numbers denote the empty vehicles. The obtained schedules for commodity dispatching moving injured people and repositioning of vehicles in the first six periods are depicted in Fig. 3. As an example, consider node N_4 at $t = 1$. At this time, four ambulances are shipped to node N_7, one helicopter is sent to node N_1 and two helicopters are sent to node N_{11}. At time 3, one of the ambulances arriving at node N_7 picks up two injured people type H_1, and carries them to the hospital located at node N_4. The remaining ambulances move to node N_9. After arriving at node N_9($t = 4$), they pick up 6 injured people type H_1, and transport them to node N_8. These injured people along with 19 injured people type H_2 of node N_8 are moved by five helicopters to the hospital located at node N_{11} at $t = 5$. Then, these three ambulances come back to node N_9. After arriving at node N_9($t = 6$), they pick up 6 injured people type H_1, and moved them to node N_8 for dispatching to the hospital located at node N_{11} by helicopters. Moreover, the helicopter shipped to node N_1 picks up 19 commodities type C_2 after arriving at $t = 2$ and moves to node N_8. The helicopters, at $t = 2$ at node N_{11}, pick up 20 commodities of type C_1 and 200 commodities type C_2, and move them to node N_8. These three helicopters (two helicopters sent from N_{11} and one helicopter sent from N_1) along with three empty helicopters (sent from N_4 at $t = 2$) pick up 25 injured people type H_1 and 5 injured people type H_2 from node N_8 at $t = 3$, and transport them to the hospital located at node N_4. As the second example, consider node N_1 at $t = 1$.

Table 11
The commodities shipment in the acquired plan (value of X variables).

Time	A_1	A_2	Vehicles	Dest.
N_1				
1	29	–	$V_1 = 1$	N_8
2	–	20	$V_1 = 1$	N_8
	–	19	$V_3 = 1$	N_8
6	19	–	$V_1 = 1$	N_8
8	–	4	$V_1 = 1$	N_8
	–	25	$V_3 = 1$	N_8
11	77	–	$V_1 = 4$	N_8
19	–	19	$V_3 = 1$	N_8
N_4				
2	19 + 18	29	$V_2 = 1$	N_7
	24	23	$V_1 = 2$	N_7
6	–	18	$V_1 = 1$	N_7
8	77	–	$V_1 = 4$	N_7
11	39	–	$V_2 = 1$	N_7
12	15	–	$V_1 = 1$	N_7
	3	–	$V_3 = 1$	N_8
15	45	–	$V_3 = 3$	N_8
	–	67	$V_1 = 2$	N_7
16	–	23	$V_2 = 1$	N_7
19	99	–	$V_1 = 4$	N_7
	–	45	$V_2 = 1$	N_7
20	29	–	$V_1 = 2$	N_7
N_8				
3	25	19 + 12	$V_1 = 2$	N_9
6	–	13 + 8	$V_1 = 1$	N_9
9	–	30	$V_1 = 1$	N_9
11	–	18	$V_1 = 1$	N_9
13	99	43	$V_1 = 7$	N_9
16	40		$V_1 = 2$	N_9
20	–	7	$V_1 = 1$	N_9
N_9				
4	–	10	$V_1 = 1$	N_7
N_{10}				
3	–	9	$V_2 = 1$	N_{11}
10	3	–	$V_1 = 1$	N_4
N_{11}				
1	18	23	$V_3 = 2$	N_4
N_{11}				
2	20	20	$V_3 = 2$	N_8
3	14	–	$V_3 = 1$	N_8
4	–	9	$V_3 = 1$	N_8
5	–	5	$V_3 = 1$	N_8
6	74	–	$V_3 = 5$	N_8
8	–	54	$V_3 = 3$	N_8
10	45	–	$V_3 = 3$	N_8
11	51	48	$V_3 = 5$	N_8
12	45	19	$V_3 = 4$	N_8
14	43	–	$V_3 = 3$	N_8
16	30	25	$V_3 = 3$	N_8
17	75	–	$V_3 = 5$	N_8
N_{12}				
4	–	19	$V_1 = 1$	N_7
11	72	–	$V_1 = 3$	N_7
N_{13}				
1	15	–	$V_3 = 1$	N_8
	14	–	$V_1 = 1$	N_{11}
	–	9	$V_1 = 1$	N_{10}
6	–	24	$V_3 = 1$	N_8
8	3	–	$V_1 = 1$	N_{10}

Table 11 (*continued*)

Time	A_1	A_2	Vehicles	Dest.
	45	–	$V_2 = 1$	N_{11}
11	–	19	$V_3 = 1$	N_{11}
15	75	–	$V_1 = 3$	N_{11}
	–	23	$V_3 = 1$	N_4
	–	25	$V_3 = 1$	N_{11}
N_{16}				
1	24	–	$V_3 = 2$	N_4
	–	19	$V_1 = 1$	N_{12}
6	–	29	$V_3 = 2$	N_8
8	15	–	$V_3 = 1$	N_8
	72	–	$V_1 = 3$	N_{12}
11	15	–	$V_3 = 1$	N_4
	43	–	$V_1 = 2$	N_{11}
15	30	–	$V_3 = 2$	N_{11}
19	29	–	$V_3 = 2$	N_4
	9	58	$V_3 = 3$	N_8

At this time, two trucks pick up 29 commodities of type C_1, and move them to node N_8. Upon arrival at node $N_8(t = 3)$, they pick up eight commodities of type C_1 and 19 commodities of type C_2 (some of the commodities were dispatched from node N_{11} by two helicopters at t = 2), and move to node N_9. At time $t = 4$,the trucks deliver 37 commodities type C_1 and 9 commodities of type C_2t node N_9. The remaining commodities (10 commodities of type C_2) are shipped to node N_7 by a single truck.

Further analysis on the obtained schedules identifies the emergency routes between demand points, suppliers and hospitals summarized in Table 13. Each arc includes two sections, the first determining the origin and destination nodes, and the second section specifying the mode of transportation. Codes 1 and 4 respectively denote transportation by truck and ambulance, code 2 indicates transportation by train and code 3 by helicopter. For instance, the emergency route for transporting commodities between supplier N_{11} and demand point N_9 includes transporting from node N_{11} to node N_8 by helicopter and from node N_8 to node N_9 by truck. Moreover, the emergency routes for transporting wounded people from this node to the hospital located at node N_4 are $(N_9–N_7, 1)–(N_7–N_4, 1)$ and $(N_9–N_8, 1)–(N_8–N_4, 3)$. In the first route, wounded people are transported by ambulance. However in the second route, the wounded people are moved from node N_9 to node N_8 by ambulance and from N_8 to node N_4 by helicopter. As it is shown in Table 14, most of these wounded people (about 90%) are transported on the second path having less transporting time. This analysis also shows that all arcs of network are not similarly important in the response because some arcs are used more for the commodity and wounded people transportation.

Table 14 shows the ten most important roads for commodity and wounded people transportation as well as weighted percentage of commodity and wounded people using these roads. As an example, consider arc $(N_{11}–N_8, 3)$ having the largest percentage of commodity transportation. This arc is part of two emergency routes: route from supplier N_{11} to the points of demand N_8 and N_9. Moreover, arc $(N_8–N_4, 3)$ existing in two emergency routes (from node N_8 to node N_4 and from node N_9 to N_4) has the highest percentage in moving wounded people. Now if the worst case occurs and road $N_9–N_7$ and $N_9–N_8$ are destroyed, the current plan would be infeasible because several scheduled movements could not be performed.

Under these circumstances, the model should be re-optimized based on the newly obtained information on the transportation network availability. These new emergency routes and ten most important roads are shown in Tables 15 and 16 respectively.

The emergency routes are shown in bold in Table 15 and differ from those obtained for the base scenario. For instance, since the commodity emergency route between nodes N_{11} and N_9 is changed from $(N_{11}–N_8, 3)–(N_8–N_9, 1)$ to $(N_{11}–N_{12}, 3)–(N_{11}–N_{12}, 1)$, the amount of commodity moved from node N_{11} to N_8 by helicopter $(N_{11}–N_8, 3)$ decreases from 33.96% to 22.86%. This change increases the rank of arc $(N_{11}–N_8, 3)$ from one to two. Also, a change in the emergency route between nodes N_1 and N_7 from $(N_1–N_8, 3)–(N_8–N_9, 1)–(N_9–N_7, 1)$ and $(N_1–N_8, 1)–(N_8–N_9, 1)–(N_9–N_7, 1)$ to $(N_1–N_4, 3)–(N_4–N_7, 1)$ results in an increase in commodity transport from node N_4 to node N_7 (20.62–26.81%), and changes its rank from three to one. Similar effects can be observed for moving the injured. For example, emergency route for transporting wounded people between nodes N_9 and N_4 changes from $(N_9–N_7, 1)–(N_7–N_4, 1)$ and $(N_9–N_8, 1)–(N_8–N_4, 3)$ to $(N_9–N_{12}, 1)–(N_{12}–N_4, 3)$. As a result, the percentage of wounded people moved from arc $(N_8–N_4, 3)$ decreases from 31.33% to 20.30%, and its rank changes from one to three. Moreover, since the wounded people dispatched to hospital located at node N_{16} often pass through node N_{12}, its importance in the disaster relief effort increases (increase of the fraction of wounded people moving from $(N_{12}–N_{16},3)$ from 2.44% to 17.39% and a new importance rank of 4).

As mentioned earlier, only half the uncertain parameters are allowed to exhibit their worst changes (that is $\Gamma_i^t = 0.5\tau_i^t$). Therefore, it is expected that the transportation plans do not use the maximum anticipated capacities of the network entities. In this case, it is expected that these plans use at most half of variable segments of capacities. Along the same lines, at most

Table 12
The vehicles movement in the acquired plan (value of Z variables).

Time	Vehicles	Dst.
N_1		
1	$V_1 = 2$	N_8
2	$V_1 = 1$, $V_3 = 1$	N_8
3	$V_2 = 1$	N_3
6	$V_1 = 1$	N_8
8	$V_1 = 1$, $V_3 = 1$	N_8
11	$V_1 = 4$	N_8
19	$V_3 = 1$	N_8
N_3		
8	$V_1 = 5$	N_1
N_4		
1	$V_3 = 1$	N_1
	$V_4 = 4$	N_7
	$V_3 = 2$	N_{11}
2	$V_1 = 2$, $V_2 = 1$	N_7
	$V_3 = 3$	N_8
	$V_3 = 1$	N_{11}
3	$V_4 = 10$	N_7
4	$V_4 = 4$	N_7
	$V_3 = 4$	N_8
	$V_3 = 2$	N_{11}
5	$V_4 = 7$	N_7
	$V_3 = 1$	N_8
6	$V_1 = 1$, $V_4 = 1$	N_7
7	$V_3 = 1$	N_1
	$V_1 = 2$	N_3
	$V_4 = 10$	N_7
8	$V_1 = 4$, $V_4 = 4$	N_7
	$V_3 = 2$	N_8
9	$V_4 = 6$	N_7
	$V_3 = 1$	N_8
10	$V_4 = 1$	N_7
	$V_3 = 1$	N_{11}
	$V_3 = 1$	N_{13}
	$V_3 = 1$	N_{16}
11	$V_2 = 1$, $V_4 = 7$	N_7
12	$V_1 = 1$, $V_4 = 4$	N_7
	$V_3 = 1$	N_8
13	$V_4 = 4$	N_7
	$V_3 = 2$	N_8
	$V_3 = 2$	N_{11}
14	$V_4 = 3$	N_{10}
	$V_3 = 1$	N_{13}
	$V_3 = 2$	N_{16}
15	$V_1 = 2$, $V_4 = 6$	N_7
	$V_3 = 3$	N_8
	$V_4 = 1$	N_{10}
16	$V_2 = 1$, $V_4 = 1$	N_7
	$V_3 = 2$	N_{11}
17	$V_4 = 9$	N_7
N_{13}		
6	$V_3 = 1$	N_8
8	$V_1 = 1$	N_{10}
	$V_2 = 1$	N_{11}
11	$V_3 = 1$	N_{11}
15	$V_3 = 1$	N_4
N_4		
18	$V_3 = 3$	N_{16}
19	$V_1 = 4$,$V_2 = \mathbf{1}$	N_7
20	$V_3 = 1$	N_5
	$\mathbf{V}_1 = \mathbf{2}$,$V_2 = 1$	N_7
22	$V_3 = 4$	N_5
	$V_3 = 3$	N_{16}

Table 12 (*continued*)

Time	Vehicles	Dst.
$\mathbf{N_5}$		
1	$V_4 = 5$	N_4
N_4	$V_4 = 2$	N_4
$\mathbf{N_7}$		
3	$\underline{V_4 = 1}$	N_4
	$V_4 = 3$	N_9
5	$V_1 = 3$	N_3
	$\underline{V_4 = 10}$	N_4
6	$\underline{V_4 = 4}$	N_4
7	$\underline{V_4 = 6}$	N_4
	$V_4 = 1$	N_9
8	$\underline{V_4 = 1}$	N_4
9	$\underline{V_4 = 7}$	N_4
	$V_4 = 3$	N_9
	$V_1 = 2$	12
10	$\underline{V_4 = 4}$	N_4
11	$\underline{V_4 = 6}$	N_4
12	$\underline{V_4 = 1}$	N_4
13	$\underline{V_4 = 7}$	N_4
14	$V_2 = 1, \underline{V_4 = 4}$	N_4
15	$\underline{V_4 = 6}$	N_4
16	$V_1 = 4$	N_4
17	$V_2 = 1, \underline{V_4 = 5}$	N_4
18	$V_2 = 1$	N_4
19	$\underline{V_4 = 11}$	N_4
	$V_4 = 7$	N_9
$\mathbf{N_8}$		
3	$\underline{V_3 = 6}$	N_4
	$\mathbf{V_1 = 2}$	N_9
	$\underline{V_3 = 1}$	N_{11}
4	$\underline{V_3 = 1}$	16
5	$V_4 = 3$	N_9
	$\underline{V_3 = 5}$	N_{11}
6	$\underline{V_3 = 1}$	N_4
	$\mathbf{V_1 = 1}$	N_9
	$\underline{V_3 = 1}$	N_{16}
7	$\underline{V_3 = 2}$	N_4
	$V_4 = 3$	N_9
	$\underline{V_3 = 3}$	N_{11}
8	$\underline{V_3 = 1}$	N_4
	$V_4 = 4$	N_9
	$\underline{V_3 = 2}$	N_{11}
9	$\underline{V_3 = 3}$	N_4
$\mathbf{N_{13}}$		
15	$\mathbf{V_1 = 3, V_3 = 1}$	N_{11}
$\mathbf{N_{16}}$		
1	$\mathbf{V_3 = 2}$	N_4
	$\mathbf{V_1 = 1}, V_4 = 4$	N_{12}
5	$V_3 = 1$	N_{13}
$\mathbf{N_8}$		
9	$\mathbf{V_1 = 1}, V_4 = 4$	N_9
	$\underline{V_3 = 3}$	N_{11}
10	$V_4 = 3$	N_9
	$\underline{V_3 = 4}$	N_{11}
11	$\mathbf{V_1 = 1}, V_4 = 8$	N_9
	$\underline{V_3 = 3}$	N_{11}
12	$\underline{V_3 = 4}$	N_4

(*continued on next page*)

Table 12 (*continued*)

Time	Vehicles	Dst.
	$V_4 = 3$	N_9
	$V_3 = 1$	N_{16}
13	$V_3 = 3$	N_4
	$V_1 = 7$, $V_4 = 9$	N_9
	$V_3 = 1$	N_{11}
14	$V_3 = 3$	N_4
	$V_4 = 2$	N_9
15	$V_3 = 1$	N_4
	$V_4 = 7$	N_9
	$V_3 = 2$	N_{11}
16	**$V_1 = 2$**, $V_4 = 3$	N_9
	$V_3 = 3$	N_{11}
17	$V_3 = 1$	N_4
	$V_4 = 7$	N_9
	$V_3 = 2$	N_{16}
18	$V_3 = 1$	N_1
	$V_4 = 3$	N_9
	$V_3 = 1$	N_{11}
19	$V_3 = 1$	N_4
	$V_3 = 1$	N_5
	$V_3 = 1$	N_{11}
20	$V_3 = 1$	N_4
	$V_3 = 1$	N_5
	$V_1 = 1$	N_9
21	$V_3 = 3$	N_4
N_9		
4	**$V_1 = 1$**	N_7
	$V_4 = 3$	N_8
6	$V_4 = 3$	N_8
7	$V_4 = 4$	N_8
8	$V_4 = 4$	N_8
9	$V_4 = 4$	N_8
10	$V_4 = 7$	N_8
11	$V_4 = 3$	N_8
12	$V_1 = 4, V_4 = 9$	N_8
13	$V_4 = 2$	N_8
14	$V_4 = 2$	N_7
	$V_4 = 7$	N_8
15	$V_4 = 3$	N_8
16	$V_4 = 7$	N_8
17	$V_1 = 1$, $V_4 = 3$	N_8
18	$V_4 = 7$	N_7
6	**$V_3 = 2$**	N_8
	$V_4 = 1$	N_{12}
7	$V_1 = 1$	N_{11}
8	**$V_3 = 1$**	N_8
	$V_1 = 3$	N_{12}
11	**$V_3 = 1$**	N_4
N_9		
19	$V_4 = 3$	N_8
20	$V_4 = 7$	N_8
N_{10}		
1	$V_4 = 5$	N_4
2	$V_4 = 4$	N_4
3	$V_4 = 2$	N_4
	$V_2 = 1$	N_{11}
10	**$V_1 = 1$**	N_4
16	$V_4 = 3$	N_{11}
17	$V_4 = 1$	N_{11}
18	$V_4 = 2$	N_{11}
19	$V_4 = 2$	N_{11}

Table 12 (*continued*)

Time	Vehicles	Dst.
N_{11}		
1	$\mathbf{V_3 = 2}$	N_4
	$V_4 = 4$	N_{10}
2	$\mathbf{V_3 = 2}$	N_8
	$V_4 = 2$	N_{10}
3	$\mathbf{V_3 = 1}$	N_8
4	$\mathbf{V_3 = 1}$	N_8
5	$\mathbf{V_3 = 1}$	N_8
	$V_3 = 1$	N_{16}
6	$\mathbf{V_3 = 5}$	N_8
8	$\mathbf{V_3 = 3}$	N_8
9	$V_3 = 2$	N_8
10	$\mathbf{V_3 = 3}$	N_8
11	$\mathbf{V_3 = 5}$	N_8
12	$\mathbf{V_3 = 4}$	N_8
13	$V_1 = 3$	N_{13}
14	$\mathbf{V_3 = 3}$	N_8
16	$\mathbf{V_3 = 3}$	N_8
	$V_3 = 2$	N_{16}
17	$\mathbf{V_3 = 5}$	N_8
	$V_4 = 3$	N_{10}
	$V_1 = 9$	N_{13}
18	$V_4 = 1$	N_{10}
19	$V_3 = 1$	N_8
19	$V_4 = 2$	N_{10}
20	$V_3 = 1$	N_4
	$V_4 = 2$	N_{10}
N12		
4	$\mathbf{V_1 = 1}$	N_7
	$V_4 = 4$	N_9
9	$V_4 = 1$	N_9
11	$\mathbf{V_1 = 3}$	N_7
16	$V_1 = 2$	N_4
N13		
1	$\mathbf{V_3 = 1}$	N_8
	$\mathbf{V_1 = 1}$	N_{10}
	$\mathbf{V_1 = 1}$	N_{11}
11	$\mathbf{V_1 = 2}$	N_{11}
14	$V_3 = 1$	N_{13}
15	$\mathbf{V_3 = 2}$	N_{11}
17	$V_3 = 2$	N_4
19	$\mathbf{V_3 = 2}$	N_4
	$\mathbf{V_3 = 3}$	N_8

half of uncertain segment of demands $(\widehat{A})$ will be considered in schedule. A careful examination of Tables 8–11 confirms these anticipations. For instance, Fig. 4 depicts remaining inventory for commodities C_1 and C_2 at node N_1 and N_{13} during the planning horizon. As Fig. 4 shows, the commodity inventory is at least half of cumulative uncertain supplies. As an illustration, consider the cumulative uncertain supply of commodity C_2 at node N_1 in Fig. 4. The remaining inventory is 1 until $t = 7$, 2 from $t = 8$ until $t = 18$, and 3 from $t = 19$ until the end of the planning horizon. (These inventories are clearly more than or equal to the half of cumulative uncertain supply $\left(\sum_{a,l}\widehat{A}_{a,l}\right)$ which is 2 until $t = 7$, 4 from $t = 8$ until $t = 18$, and 5 from $t = 19$ until the end of the planning horizon. Note that, these inventory values are calculated based on the difference between the maximum of available supply (sum of deterministic supply and uncertain supply) and dispatched commodities from that supplier.

Fig. 5 depicts the hospitals' remaining capacities at nodes N_9 and N_{11} during the planning horizon. Although most of the injured people were sent to these hospitals, their remaining capacity does not become zero in any period, and at least half of their cumulative uncertain capacity is unused. For example, the cumulative uncertain capacity for treating injuries of type H_1 at node $N_1\left(\sum_t\widehat{H}_{1lt}\right)$ is 2 until $t = 4$, is 4 from $t = 5$ until $t = 11$, and is 6 from $t = 12$ until the end of planning horizon. As Fig. 5 shows, the unused capacity for treating injury type H_1 at the hospital located at N_1 is high in the first periods, is 2 at

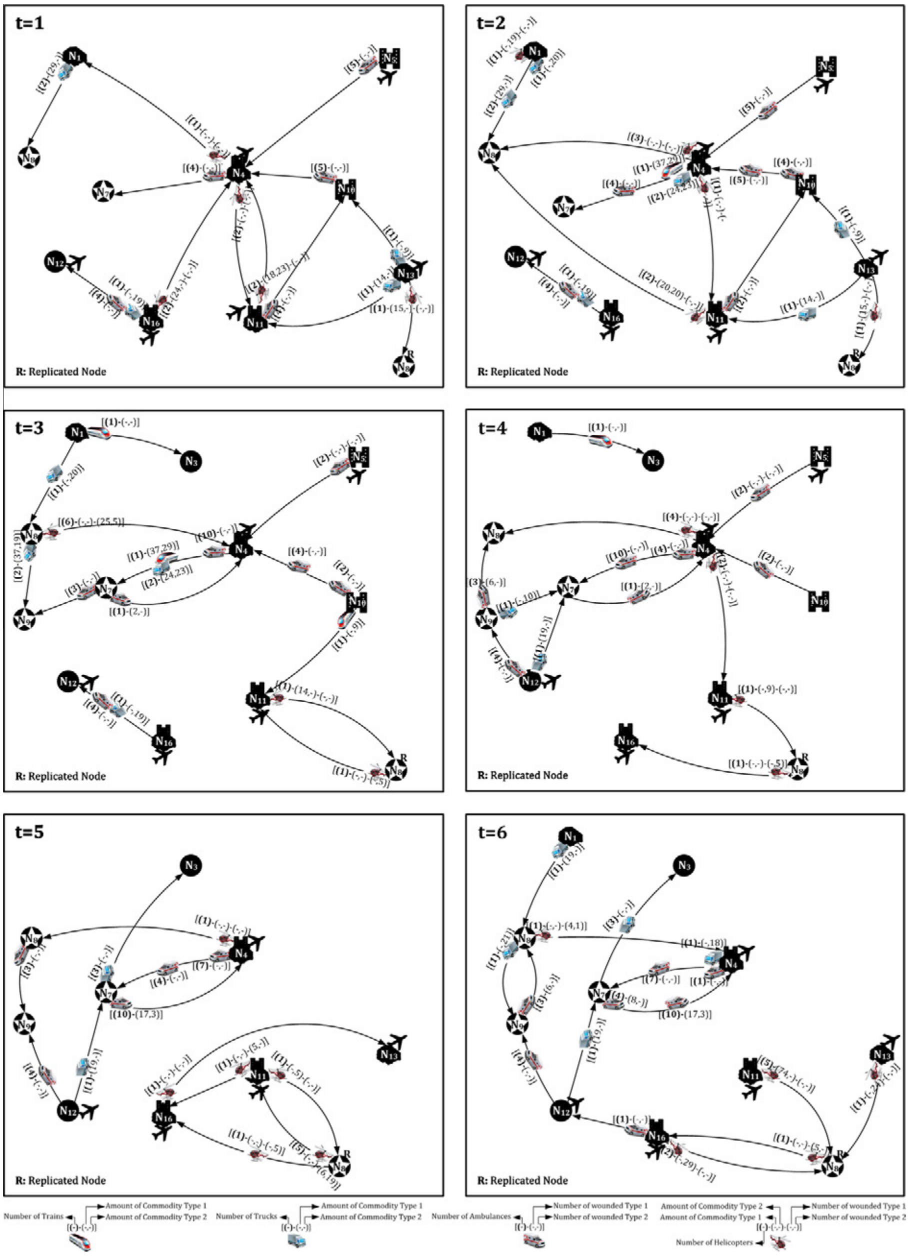

Fig. 3. Logistics activities during the first six periods.

$t = 11$ (which is half of cumulative uncertain capacity at $t = 11$), is also high from $t = 12$ until $t = 20$, and is 3 from $t = 21$ until the end of planning horizon. In other words, the used capacity is also half of cumulative uncertain capacity from $t = 21$ until $t = 24$.

Finally it is worth noting that, the proposed solution methodology is developed for problems which are moderately complex (usually: 20–25 nodes, 100–150 transportation arcs and 150–200 different vehicles). In this case, this methodology can solve the problem in only a few minutes by GAMS and CPLEX 12.1 (see Table 9).

7. Conclusions and future research

This paper contributes to the literature by proposing a novel mathematical model for assisting disaster managers in scheduling the logistical activities for disaster relief materials and injured people in the face of demand and supply uncertainties occurring in practice, and solving it by means of a robust approach for stochastic models with uncertain right-hand sides based on Bertsimas and Sim (2004). The proposed model incorporates the key features extracted from

Table 13
Emergency routes for transportation in the main scenario.

		Point of demands		
		N$_7$	N$_8$	N$_9$
Emergency route for commodity transportation				
Suppliers	N$_1$	(N$_1$–N$_8$,3)–(N$_8$–N$_9$,1)–(N$_9$–N$_7$,1) (N$_1$–N$_8$,1)–(N$_8$–N$_9$,1)–(N$_9$–N$_7$,1)	(N$_1$–N$_8$,1) (N$_1$–N$_8$,3)	(N$_1$–N$_8$,1)–(N$_8$–N$_9$,1) (N$_1$–N$_8$,3)–(N$_8$–N$_9$,1)
	N$_4$	(N$_4$–N$_7$,1) (N$_4$–N$_7$,2)	(N$_4$–N$_8$,3)	–
	N$_{11}$	(N$_{11}$–N$_4$,3)–(N$_4$–N$_7$,1) (N$_{11}$–N$_4$,3)–(N$_4$–N$_7$,2)	(N$_{11}$–N$_8$,3)	(N$_{11}$–N$_8$,3)–(N$_8$–N$_9$,1)
	N$_{13}$	(N$_{13}$–N$_4$,3)–(N$_4$–N$_7$,2)	(N$_{13}$–N$_8$,3)	(N$_{13}$–N$_8$,3)–(N$_8$–N$_9$,1)
	N$_{16}$	(N$_{16}$–N$_{12}$,1)–(N$_{12}$–N$_7$,1) (N$_{16}$–N$_4$,3)–(N$_4$–N$_7$,1)	(N$_{16}$–N$_8$,3)	–
Emergency route for wounded people transportation				
Hospitals	N$_4$	(N$_7$–N$_4$,1)	(N$_8$–N$_4$,3)	(N$_9$–N$_7$,1)–(N$_7$–N$_4$,1) (N$_9$–N$_8$,1)–(N$_8$–N$_4$,3)
	N$_5$	(N$_7$–N$_4$,1)–(N$_4$–N$_5$,3)	(N$_8$–N$_5$,3)	(N$_9$–N$_8$,1)–(N$_8$–N$_5$,3)
	N$_{10}$	–	–	(N$_9$–N$_8$,1)–(N$_8$–N$_{11}$,3)–(N$_{11}$–N$_{10}$,1)
	N$_{11}$	–	(N$_8$–N$_{11}$,3)	(N$_9$–N$_8$,1)–(N$_8$–N$_{11}$,3)
	N$_{16}$	(N$_7$–N$_4$,1)–(N$_4$–N$_{16}$,3)	(N$_8$–N$_{16}$,3)	(N$_9$–N$_{12}$,1)–(N$_{12}$–N$_{16}$,3) (N$_9$–N$_8$,1)–(N$_8$–N$_{16}$,3)

Table 14
The most important roads in the main scenario.

Commodity transportation					
Rank	1	2	3	4	5
Arc	(N$_{11}$–N$_8$,3)	(N$_8$–N$_9$,1)	(N$_4$–N$_7$,1)	(N$_4$–N$_7$,2)	(N$_{16}$–N$_8$,3)
% Commodity	33.96%	21.37%	20.62%	12.57%	9.44%
Rank	6	7	8	9	10
Arc	(N$_1$–N$_8$,1)	(N$_1$–N$_8$,3)	(N$_{12}$–N$_7$,1)	(N$_{16}$–N$_{12}$,1)	(N$_{13}$–N$_{11}$,3)
% Commodity	7.52%	6.11%	4.84%	4.84%	4.27%
Wounded people transportation					
Rank	1	2	3	4	5
Arc	(N$_8$–N$_4$,3)	(N$_8$–N$_{11}$,3)	(N$_7$–N$_4$,4)	(N$_9$–N$_8$,4)	(N$_4$–N$_{16}$,3)
% Wounded	31.33%	31.19%	30.40%	26.45%	7.85%
Rank	6	7	8	9	10
Arc	(N$_4$–N$_5$,3)	(N$_{11}$–N$_{16}$,3)	(N$_9$–N$_7$,4)	(N$_{11}$–N$_{10}$,4)	(N$_8$–N$_{12}$,3)
% Wounded	3.42%	3.14%	2.93%	2.60%	2.44%

Table 15
Emergency routes for transportation in the worst case scenario.

		Point of demands		
		N$_7$	**N$_8$**	**N$_9$**
Emergency route for commodity transportation				
Suppliers	**N$_1$**	**(N$_1$–N$_4$,3)–(N$_4$–N$_7$,1)**	(N$_1$–N$_8$,1) (N$_1$–N$_8$,3)	**(N$_1$–N$_8$,3)–(N$_8$–N$_{12}$,3)–(N$_{12}$–N$_9$,1)** **(N$_1$–N$_8$,1)–(N$_8$–N$_{12}$,3)–(N$_{12}$–N$_9$,1)**
	N$_4$	(N$_4$–N$_7$,1) (N$_4$–N$_7$,2)	(N4–N8,3)	–
	N$_{11}$	(N$_{11}$–N$_4$,3)–(N$_4$–N$_7$,1)	(N$_{11}$–N$_8$,3)	**(N$_{11}$–N$_{12}$,3)–(N$_{12}$–N$_9$,1)**
	N$_{13}$	(N$_{13}$–N$_4$,3)–(N$_4$–N$_7$,1) (N$_{13}$–N$_4$,3)–(N$_4$–N$_7$,2)	(N$_{13}$–N$_8$,3)	**(N$_{13}$–N$_{12}$,3)–(N$_{12}$–N$_9$,1)**
	N$_{16}$	(N$_{16}$–N$_4$,3)–(N$_4$–N$_7$,1)	(N$_{16}$–N$_8$,3)	**(N$_{16}$–N$_{12}$,1)–(N$_{12}$–N$_9$,1)** **(N$_{16}$–N$_{12}$,3)–(N$_{12}$–N$_9$,1)**
Emergency route for wounded people transportation				
Hospitals	**N$_4$**	(N$_7$–N$_4$,1)	(N$_8$–N$_4$,3)	**(N$_9$–N$_{12}$,1)–(N$_{12}$–N$_4$,3)**
	N$_5$	–	–	–
	N$_{10}$	–	–	–
	N$_{11}$	–	(**N$_8$–N$_{11}$**,3)**(N$_8$–N$_{12}$,3)–(N$_{12}$–N$_{11}$,1)**	**(N$_9$–N$_{12}$,1)–(N$_{12}$–N$_{11}$,1)**
	N$_{16}$	**(N$_7$–N$_{12}$,1)–(N$_{12}$–N$_{16}$,3)**	–	**(N$_9$–N$_{12}$,1)–(N$_{12}$–N$_{16}$,3)**

Table 16
The most important roads in the main scenario.

Commodity transportation					
Rank	1	2	3	4	5
Arc	(N_4–N_7,1)	(N_{11}–N_8,3)	(N_{12}–N_9,1)	(N_4–N_7,2)	(N_1–N_8,1)
% Commodity	26.81%	22.86%	20.03%	12.53%	8.98%
Rank	6	7	8	9	10
Arc	(N_{16}–N_{12},3)	(N_{16}–N_{12},1)	(N_{13}–N_8,3)	(N_{16}–N_4,3)	(N_{13}–N_4,3)
% Commodity	8.08%	5.46%	5.41%	5.30%	4.20%
Wounded people transportation					
Rank	1	2	3	4	5
Arc	(N_8–N_{11},3)	(N_7–N_4,4)	(N_8–N_4,3)	(N_{12}–N_{16},3)	(N_9–N_{12},4)
% Wounded	28.72%	28.59%	20.30%	17.39%	16.63%
Rank	6	7	8	9	10
Arc	(N_7–N_{12},4)	(N_{12}–N_4,3)	(N_{12}–N_{11},3)	(N_1–N_4,3)	(N_8–N_1,3)
% Wounded	3.84%	2.72%	1.32%	0.96%	0.96%

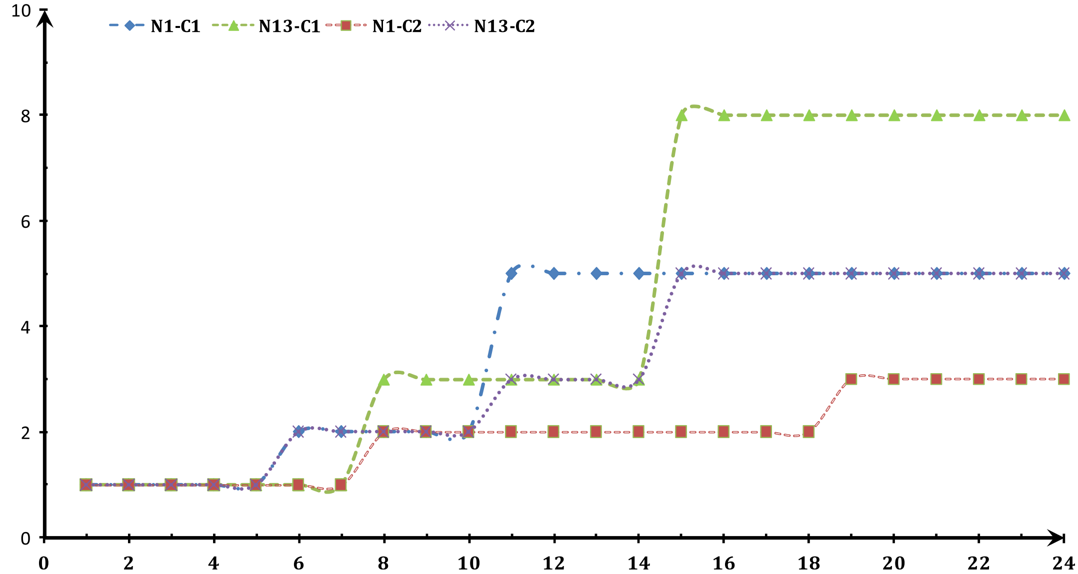

Fig. 4. Remaining inventory for commodities C_1 and C_2 at nodes N_1 and N_{13}.

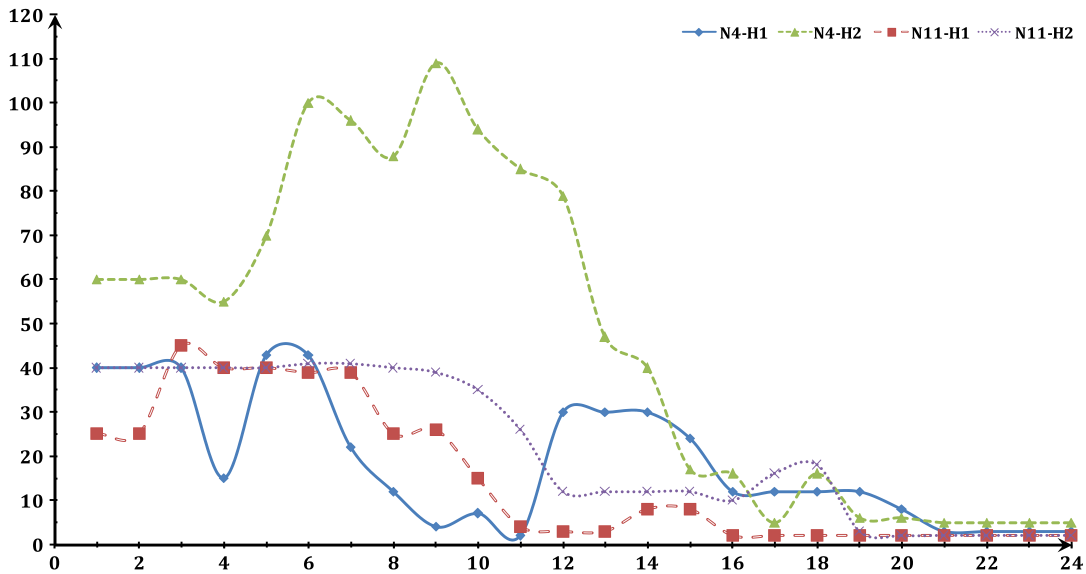

Fig. 5. Hospitals' remaining capacity at nodes N_4 and N_{11}.

previous studies, as summarized in Table 2. In particular, the proposed linear model includes three objective functions to incorporate humanitarian and cost issues in managing both disaster relief commodities and injured people during the initial phase of earthquake response. In addition, this model is optimized for emergency relief conditions under capacity and demand uncertainty. Finally, this model is capable of routing and designing solutions for different capacitated transport modes, including combined transport, which cannot be generated by previous models.

Human disaster planners want to serve as many injured people and respond to the requests for disaster relief materials as quickly as possible during the emergency response phase with the lowest possible number of vehicles. Therefore an equivalent multi objective stochastic model is presented that is capable of handling real-life uncertainties in an intuitively appealing way. Disaster response planners look for schedules that are robust in the face of demand and supply uncertainty. Hence, a robust model formulation was developed and the method of Bertsimas and Sim (2004) was modified to produce the linear robust counterpart of the stochastic model. A solution methodology was suggested for solving the acquired robust counterpart of the stochastic model. This methodology converts the main model to three sub-models, and uses three steps to optimize the three objectives of SRMLM model hierarchically.

Finally, an illustrative example was presented to show how the model is capable of capturing all crucial network information and how the solution methodology generates robust solutions in acceptable computation times.

Further research will be aimed at optimizing the network structure which has been considered fixed in the current paper and developing dynamic scheduling algorithms. Network data such as travel time, capacities and demands are uncertain, but may also change over time. Moreover, natural disasters such as earthquake may destroy roads and which further increases the need for dynamic scheduling based on real-time information. Therefore, developing an online scheduling model capable of processing real-time data in planning logistics activities could significantly improve the disaster response process. Although most problems occurring in practice can be solved by the proposed methodology, one may encounter very large problem instances for which significantly larger computation times are required. Meta-heuristic solution approaches for the proposed model therefore offer an interesting alternative to tackle these instances and to support dynamic decision-making.

Appendix A

Proposition 1. *The robust counterpart of model (A.1)*

$$\begin{aligned} \text{Min} \quad & z=\sum_j c_j x_j, \\ & \sum_j a_{ij}x_j \leqslant \tilde{b}_i \quad \forall i, \\ & x_j \geqslant 0 \quad \forall j, \end{aligned} \tag{A.1}$$

is model (A.2).

$$\begin{aligned} \text{Min} \quad & z=\sum_j c_j x_j, \\ \text{s.t.}: \quad & \sum_j a_{ij}x_j + z_i\Gamma_i + \sum_{s\in\tau_i} p_{is} \leqslant b_i \quad \forall i, \\ & z_i + p_{is} \geqslant \hat{b}_{is} \quad \forall i, s\in\tau_i, \\ & x_j \geqslant 0 \quad \forall j, \\ & p_{is} \geqslant 0 \quad \forall i, s\in\tau_i, \\ & z_i \geqslant 0 \quad \forall i. \end{aligned} \tag{A.2}$$

Proof. As mentioned in Section 3, since the acquired solution should be feasible in the worst case of defined condition, $\beta(\tau_i,\Gamma_i)$ would be defined as follows:

$$\beta(\tau_i,\Gamma_i)=\underset{\{S_i\cup t_i|S_i\subseteq\tau_i,|S_i|=\lfloor\Gamma_i\rfloor,t_i\in\tau_i\setminus S_i\}}{\text{Max}}\left\{\sum_{s\in\tau_i}\hat{b}_{is}+(\Gamma_i-\lfloor\Gamma_i\rfloor)\hat{b}_{it_i}\right\}, \tag{A.3}$$

and the related non-linear robust optimization model is,

$$\begin{aligned} \text{Max} \quad & w=\sum_j -c_j x_i, \\ & \sum_j a_{ij}x_{ij} + \underset{\{S_i\cup t_i|S_i\subseteq\tau_i,|S_i|=\lfloor\Gamma_i\rfloor,t_i\in\tau_i\setminus S_i\}}{\text{Max}}\left\{\sum_{s\in\tau_i}\hat{b}_{is}+(\Gamma_i-\lfloor\Gamma_i\rfloor)\hat{b}_{it_i}\right\} \leqslant b_i \quad \forall i, \\ & x_j \geqslant 0 \quad \forall i,j. \end{aligned} \tag{A.4}$$

Although all parameters of this proposed model (A.4) are deterministic, it is a nonlinear model. To convert this model to a linear model, the linear equivalent of $\beta(\tau_i, \Gamma_i)$ would bedefined. Obviously, $\beta(\tau_i, \Gamma_i)$ in model (A.4) equals to the following linear optimization model:

$$\begin{aligned} &\beta(\tau_i, \Gamma_i) = Max \sum_{s \in \tau_i} \hat{b}_{is} \zeta_{is}, \\ &\sum_{s \in \tau_i} \zeta_{is} \leqslant \Gamma_i, \\ &0 \leqslant \zeta_{is} \leqslant 1 \quad \forall s. \end{aligned} \tag{A.5}$$

Now, consider the dual of model (A.5), which is as follows,

$$\begin{aligned} \text{Min} \quad & Z' = \sum_{s \in \tau_i} p_{is} + z_i \Gamma_i, \\ & z_i + p_{is} \geqslant \hat{b}_{is} \quad \forall i, s \in \tau_i, \\ & p_{is} \geqslant 0 \quad \forall i, s \in \tau_i, \\ & z_i \geqslant 0 \quad \forall i. \end{aligned} \tag{A.6}$$

According to the strong duality theorem, since model (A.5) is feasible and bounded for all $\Gamma_i \in [0, |\tau_i|]$, then the dual model (A.6) is also feasible, bounded and their optimal objective functions are equal. Thus,

$$\beta^*(\tau_i, \Gamma_i) = \sum_{s \in \tau_i} p^*_{is} + z^*_i \Gamma_i, \tag{A.7}$$

where, $z^*_i + p^*_{is} \geqslant \hat{b}_{is}$. Now, by replacing model (A.6) in model (A.4), model (A.8) is obtained.

$$\begin{aligned} \text{Min} \quad & z == \sum_j c_j x_j, \\ \text{s.t.}: \quad & \sum_j a_{ij} x_j + z_i \Gamma_i + \sum_{s \in \tau_i} p_{is} \leqslant b_i \quad \forall i, \\ & z_i + p_{is} \geqslant \hat{b}_{is} \quad \forall i, s \in \tau_i, \\ & x_j \geqslant 0 \quad \forall j, \\ & p_{is} \geqslant 0 \quad \forall i, s \in \tau_i, \\ & z_i \geqslant 0 \quad \forall i. \qquad \square \end{aligned} \tag{A.8}$$

Proposition 2. *An optimal solution of model (A.8) is feasible for model (A.1) if up to $\lfloor \Gamma_i \rfloor$ parameters of ith right-hand side change and one parameter $\tilde{b}_{it_i}$ changes by $(\Gamma_i - \lfloor \Gamma_i \rfloor)\hat{b}_{it_i}$.*

Proof. Let us assume that x^*_{ij} is the optimal solution of model (A.8). According to its constraint

$$\sum_j a_{ij} x^*_{ij} + \sum_{s \in \tau_i} p^*_{is} + z^*_i \Gamma_i \leqslant \sum_{s \in \tau_i} b_{is} = b_i. \tag{A.9}$$

Since $\sum_{s \in \tau_i} p^*_{is} + z^*_i \Gamma_i = \beta^*(\tau_i, \Gamma_i)$, Eq. (A-9) could be rewritten as follows,

$$\sum_j a_{ij} x^*_{ij} + \beta^*(\tau_i, \Gamma_i) \leqslant b_i \Rightarrow \sum_j a_{ij} x^*_{ij} \leqslant b_i - \beta^*(\tau_i, \Gamma_i). \tag{A.10}$$

Now, presume that $\lfloor \Gamma_i \rfloor$ parameters of ith right-hand side change. In the worst case, all changes are in the negative direction. That is, for $\lfloor \Gamma_i \rfloor$ changed parameters

$$\tilde{b}_{il} = b_{il} - \hat{b}_{il} \quad l \in \tau'_i \subseteq \tau_i, |\tau'_i| = \lfloor \Gamma_i \rfloor, \tag{A.11}$$

and for the parameter $\tilde{b}_{it_i}$,

$$\tilde{b}_{it_i} = b_{it_i} - (\Gamma_i - \lfloor \Gamma_i \rfloor)\hat{b}_{it_i}. \tag{A.12}$$

Therefore, $\tilde{b}_i$ could be calculated as follows:

$$\tilde{b}_i = \sum_{s\in\tau_i}\tilde{b}_{is} = \sum_{s\in\tau_i\setminus\tau_i'\cup t_i} b_{is} + \sum_{l\in\tau_i'}(b_{il} - \hat{b}_{il}) + (b_{it_i} - (\Gamma_i - \lfloor\Gamma_i\rfloor)\hat{b}_{it_i}),$$
$$= \sum_{s\in\tau_i\setminus\tau_i'\cup t_i} b_{is} + \sum_{l\in\tau_i'} b_{il} - \sum_{l\in\tau_i'}\hat{b}_{il} + (b_{it_i} - (\Gamma_i - \lfloor\Gamma_i\rfloor)\hat{b}_{it_i}) = \left(\sum_{s\in\tau_i\setminus\tau_i'\cup t_i} b_{is} + \sum_{l\in\tau_i'} b_{il} + b_{it_i}\right) - \left(\sum_{l\in\tau_i'}\hat{b}_{il} + (\Gamma_i - \lfloor\Gamma_i\rfloor])\hat{b}_{it_i}\right),$$
$$= \sum_{s\in\tau_i} b_{is} - \left(\sum_{l\in\tau_i'}\hat{b}_{il} + (\Gamma_i - \lfloor\Gamma_i\rfloor)\hat{b}_{it_i}\right) = \sum_{s\in\tau_i} b_{is} - \hat{\theta}_i. \tag{A.13}$$

Now, according to Eqs. (A.3) and (A.13),

$$\beta^*(\tau_i, \Gamma_i) = \underset{\{S_i\cup t_i \mid |S_i| = \lfloor\Gamma_i\rfloor, t_i\in\tau_i\setminus S_i\}}{Max}\left\{\sum_{s\in S_i}\hat{b}_{is} + (\Gamma_i - \lfloor\Gamma_i\rfloor)\hat{b}_{it_i}\right\}, \tag{A.14}$$

which is always bigger than or equals to $\hat{\theta}_i$. So, we have

$$b_i - \hat{\theta}_i \geqslant b_i - \beta^*(\tau_i, \Gamma_i) = \tilde{b}_i. \tag{A.15}$$

Now, Eqs. (A.9) and (A.15) demonstrate that x_{ij}^* is a feasible solution of model (24). □

Proposition 3. *If more than $\lfloor\Gamma_i\rfloor$ parameters of ith right-hand side change, the probability of the ith constraint violation is less than or equals to $B(n, \Gamma_i)$ where,*

$$B(n, \Gamma_i) = \frac{1}{2^n}\left\{(1-\mu)\binom{n}{\lfloor v\rfloor} + \sum_{l=\lfloor v\rfloor+1}^{n}\binom{n}{l}\right\}, \tag{A.16}$$

where, $n = |\tau_i| v = (\Gamma_i + n)/2$ and $\mu = v - \lfloor v\rfloor$.

Proof. Let us assume that x_{ij}^*, S_i^* and t_i^* are the optimal solutions of model (A.8). So, the probability of violation of *i*th constraint could be calculated as follows:

$$P\left(\sum_j a_{ij}x_{ij}^* > \tilde{b}_i\right) = P\left(\sum_j a_{ij}x_{ij}^* > \sum_{s\in\tau_i}\tilde{b}_{is}\right). \tag{A.17}$$

Also, parameter η_{is} is defined as follows:

$$\eta_{is} = \frac{b_{is} - \tilde{b}_{is}}{\hat{b}_{is}}, \tag{A.18}$$

Since the parameters $\tilde{b}_{is}$ have independent symmetric distributions, parameters η_{is} are also independent and symmetrically distributed in $[-1, 1]$. So,

$$\tilde{b}_{is} = b_{is} - \eta_{is}\hat{b}_{is}. \tag{A.19}$$

Therefore,

$$P\left(\sum_j a_{ij}x_{ij}^* > \tilde{b}_i\right) = P\left(\sum_j a_{ij}x_{ij}^* > \sum_{s\in\tau_i}(b_{is} - \eta_{is}\hat{b}_{is})\right) = P\left(\sum_j a_{ij}x_{ij}^* > \sum_{s\in\tau_i} b_{is} - \sum_{s\in\tau_i}\eta_{is}\hat{b}_{is}\right) = P\left(\sum_j a_{ij}x_{ij}^* > b_i - \sum_{s\in\tau_i}\eta_{is}\hat{b}_{is}\right). \tag{A.20}$$

Since x_{ij}^* is the optimal solution of model (A.8), then

$$\sum_j a_{ij}x_{ij}^* + \beta^*(\tau_i, \Gamma_i) \leqslant b_i. \tag{A.21}$$

So, according to Eqs. (A.17) and (A.21), it could be claimed that

$$P\left(\sum_j a_{ij}x_{ij}^* > \tilde{b}_i\right) \leqslant P\left(\sum_j a_{ij}x_{ij}^* > \sum_j a_{ij}x_{ij}^* + \beta(t,\Gamma_i) - \sum_{s\in\tau_i}\eta_{is}\hat{b}_{is}\right) = P\left(\sum_{s\in S_i^*}\hat{b}_{is} + (\Gamma_i - [\Gamma_i])\hat{b}_{it_i^*} - \sum_{s\in\tau_i}\eta_{is}\hat{b}_{is} < 0\right)$$
$$= P\left(\sum_{s\in S_i^*}(1-\eta_{is})\hat{b}_{is} + (\Gamma_i - [\Gamma_i])\hat{b}_{it_i^*} - \sum_{l\in\tau_i\setminus S_i^*}\eta_{il}\hat{b}_{il} < 0\right)$$
$$= P\left(\sum_{l\in\tau_i\setminus S_i^*}\eta_{il}\hat{b}_{il} > \sum_{s\in S_i^*}(1-\eta_{is})\hat{b}_{is} + (\Gamma_i - [\Gamma_i])\hat{b}_{it_i^*}\right). \quad (A.22)$$

Thus,

$$P\left(\sum_j a_{ij}x_{ij}^* > \tilde{b}_i\right) \leqslant P\left(\sum_{l\in\tau_i\setminus S_i^*}\eta_{il}\hat{b}_{il} > \sum_{s\in S_i^*}(1-\eta_{is})\hat{b}_{is} + (\Gamma_i - [\Gamma_i])\hat{b}_{it_i^*}\right), \quad (A.23)$$

Now, we choose $r^* = \arg\max_{s\in\tau_i}\{b_{is}\}$. So,

$$P\left(\sum_{l\in\tau_i\setminus S_i^*}\eta_{il}\hat{b}_{il} > \sum_{s\in S_i^*}(1-\eta_{is})\hat{b}_{is} + (\Gamma_i - \lfloor\Gamma_i\rfloor)\hat{b}_{it_i^*}\right) \leqslant P\left(\sum_{l\in\tau_i\setminus S_i^*}\eta_{il}\hat{b}_{il} > \hat{b}_{ir^*}\left(\sum_{s\in S_i^*}(1-\eta_{is}) + (\Gamma_i - \lfloor\Gamma_i\rfloor)\right)\right)$$
$$= P\left(\sum_{l\in\tau_i\setminus S_i^*}\eta_{il}\hat{b}_{il} > \hat{b}_{ir^*}\left(\sum_{s\in S_i^*}1 - \sum_{s\in S_i^*}\eta_{is} + (\Gamma_i - \lfloor\Gamma_i\rfloor)\right)\right)$$
$$= P\left(\sum_{l\in\tau_i\setminus S_i^*}\eta_{il}\hat{b}_{il} > \hat{b}_{ir^*}\left(\Gamma_i - \sum_{s\in S_i^*}\eta_{is}\right) = \hat{b}_{ir^*}\Gamma_i - \sum_{s\in S_i^*}\eta_{is}\hat{b}_{ir^*}\right)$$
$$= P\left(\sum_{s\in S_i^*}\eta_{is} + \sum_{l\in\tau_i\setminus S_i^*}\eta_{il}\hat{b}_{il}/\hat{b}_{ir^*} > \Gamma_i\right), \quad (A.24)$$

or

$$P\left(\sum_{s\in\tau_i}\eta_{is}\gamma_{is} > \Gamma_i\right) \leqslant P\left(\sum_{s\in\tau_i}\eta_{is}\gamma_{is} \geqslant \Gamma_i\right), \quad (A.25)$$

where

$$\gamma_{is} = \begin{cases} 1 & \text{if } s\in S_i^* \\ \hat{b}_{is}/\hat{b}_{ir^*} & \text{if } s\notin S_i^* \end{cases}. \quad (A.26)$$

Therefore, the upper limit of probability of violation in the ith constraint is as follows:

$$P\left(\sum_j a_{ij}x_{ij}^* > \tilde{b}_i\right) \leqslant P\left(\sum_{s\in\tau_i}\eta_{is}\gamma_{is} \geqslant \Gamma_i\right) \quad (A.27)$$

Finally, since parameters η_{is} are also independent random variables distributed symmetrically in $[-1,1]$; Theorem 3 in Bertsimas and Sim (2004) demonstrate that,

$$P\left(\sum_{s\in\tau_i}\eta_{is}\gamma_{is} \geqslant \Gamma_i\right) \leqslant B(n,\Gamma_i). \quad (A.28)$$

Therefore,

$$P\left(\sum_j a_{ij}x_{ij}^* > \tilde{b}_i\right) \leqslant B(n,\Gamma_i). \qquad \square \quad (A.29)$$

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